STATISTICAL MECHANICS FOR WIRELESS SYSTEMS:
APPLICATION OF EXCLUSION PROCESSES TO THE MODELING AND ANALYSIS OF MULTIHOP NETWORKS

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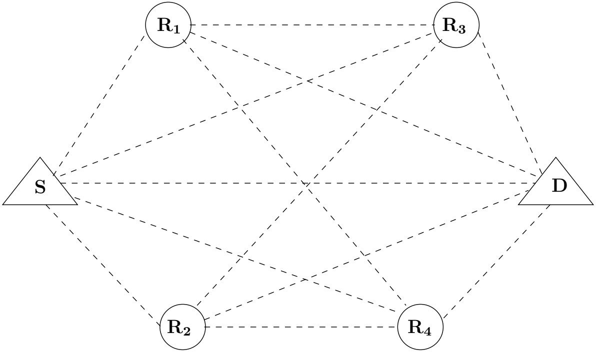
A Surge of Research Developments: CDMA, OFDM, MIMO, Turbo codes.

"Tetherless Connectivity"	single-hop	cellular, WLAN
"Ubiquitous connectivity"	multihop	multihop cellular, ad hoc



A multihop network with a single source-destination pair.

Why Multihop Wireless Networks?

Multihop networks are **highly appealing**:

- Require less per-node power.
- Operate in a decentralized fashion.
- Lack single points of failure.
- Rapidly deployable and reconfigurable.

=> facilitate 'anywhere-anytime" communication.

Immediate applications:
Battlefield networks, mesh networks, sensor networks.

Motivation

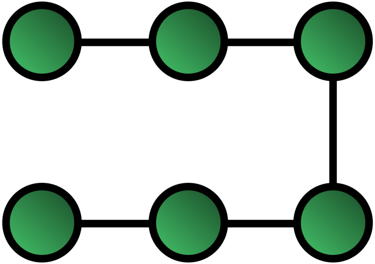
- However, the **lack of a capacity theory** capable of quantifying the performance of a general multihop network has stunted its development and commercialization [Andrews '08].
    - Classical information theory is inadequate to study multihop networks (N nodes => $(N^2 - N)$ possible one-way interactions).
    - Interdependencies between flows across various links need to be explicitly considered during analysis and design.
    - An adaptive cross-layer design approach needs to be implemented.
- Consequently, we **need to consider other subject areas** to obtain ideas and methodologies. We use a combination of tools:

Poisson Shot Noise Theory	Stochastic Geometry	well-known
Totally Asymmetric Simple Exclusion Process (TASEP)	Statistical Mechanics	unfamiliar

Outline

- Analyze the linear network topology.
  - Propose a distributed transmission policy for regulating packet flow.
  - Characterize its end-to-end delay and throughput performances.
- Study networks with more intricate topologies.
  - Consider networks with intersecting routes.
  - Propose a new framework that helps evaluate their throughput and delay.
- Investigate the throughput-delay-reliability (TDR) tradeoffs.
  - Study the delay-throughput tradeoffs at unit reliability.
  - Show that dropping a few packets can actually lead to an improved network performance.

Part 1: Throughput and Delay Analysis of Line Networks



System Model

- Nodes are located on a line with separation d.
- Time is slotted to the duration of a packet.
- Each transmitting node transmits at unit power.
- All nodes use the same channel;
- Attenuation in the channel: product of
  - Large scale path loss with exponent γ.
  - Small-scale Rayleigh fading.
- Success probability across a link: $p_s = \Pr(\text{SINR} > \Theta)$.

Prior Work

- Focuses primarily on mean delays, delay correlations are neglected [Gallucio '09, Jun '09].
- Considers very small [Ryoki '02, Daduna '00] or infinite networks [Gupta '00].
- Neglects queueing delays/ assumes infinite buffer capacities/ considers backlogged nodes [Abouei '07, Yang '03].

Our Contributions

- Advocate a revised transmission scheme to overcome current shortcomings.
- Draw analogies between packet flow in the multihop line network and the Totally Asymmetric Simple Exclusion Process (TASEP).
- Provide results that are scalable via a clean, and rigorous analysis.

Existing Transmission Policies for Line Networks

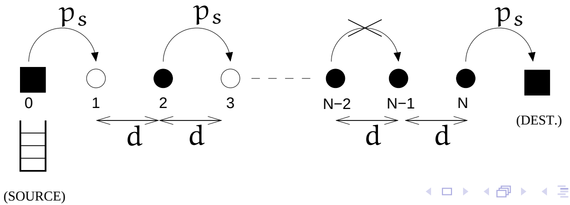
- Multihop networks are not just meant to carry small volumes of data, but may also be intended for broadband services, e.g., mesh networks.
- However, existing buffering policies with **large buffer sizes and a drop-tail policy** have inherent drawbacks: large queueing delays, non-coordinated transmissions, buffer overflows [Xu '01, Fu '03].
- Consequently, the end-to-end delay and throughput performance in such systems is disappointing.

A Revised Transmission Policy for Line Networks

The Three Rules

1. All the queueing is performed at the source node (while relay nodes have unit buffer sizes).
2. Transmissions are not accepted by nodes if their adjacent node's buffer already contains a packet.
3. Packets are retransmitted until successful reception (100% reliability).

- A Tx can occur only if node has a packet and its adjacent node none.
- This scheme is completely distributed and helps regulate the flow of packets in the network by spacing packets.

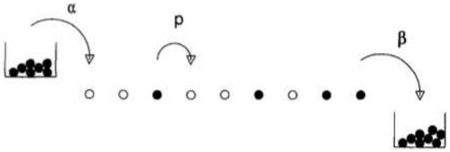


Advantages of the Single-Buffer Scheme

- Lowers average in-network delay.
  - Stacking-up of packets in buffers is minimal.
- Lessens the variance of the delay.
  - Generally requires buffering at the source. However, packet delays are more tightly controlled.
  - Depending on the time a packet spends in its buffer, the source itself can judiciously decide whether to drop it or not.
- Reduces hardware cost and energy consumption.
- Minimizes end-to-end buffer usage [Venkataramanan '10], provides buffering gain [Bhadra '06], self-organizes network operation [Dousse '07].

A Review of TASEPs with Open Boundaries

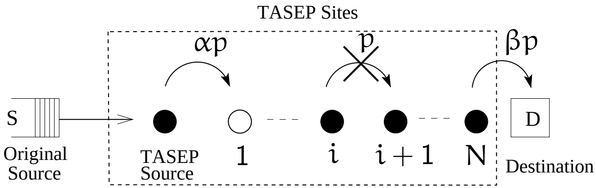
- The TASEP models the dynamics of self-driven systems with several interacting particles; is a paradigm for non-equilibrium systems .



- The source site always has a particle.
- Totally Asymmetric: Particles are injected at the leftmost site, and they hop **rightward** until they exit the system.
- Simple Exclusion Process: Each site can **have a particle or not**.
- A set of sites is picked, the particles on those sites attempt to hop.
- α and β represent the *influx* and *outflux rates*.
- Particles (filled circles) moving rightward is equivalent to *holes* (empty circles) moving leftward: particle-hole symmetry.

Note the analogies

- Sites ⇔ Relay nodes.
- Particles ⇔ Packets.
- Exclusion principle ⇔ Unit buffer sizes.
- Hopping probability ⇔ Link reliability.



TASEP-equivalent network flow.

- The configuration of site i, $1 \leqslant i \leqslant N$ at time t is $\tau_i[t] \in \{0, 1\}$.
- $\tau_0[t] = 1, \forall t \Leftrightarrow$ backlogged source.
- MAC scheme: related to the TASEP *updating procedure* - **random-sequential**, ordered-sequential, sublattice parallel, **parallel**.

r-TDMA-based Line Network

- MAC: randomized TDMA (r-TDMA): Tx node in each slot is chosen uniformly randomly (w.p. $1/(N+1)$) $\Leftrightarrow$ *random sequential* TASEP.
- Since interference is absent in the system, the success probability across each link is $p_s = \mathbb{P}(\mathsf{SNR} > \Theta) = \exp(-\Theta N_0 d^{-\gamma})$.

Metrics of Interest

- The **throughput** T, is defined as the average number of packets successfully delivered (to the destination) in unit time.
- The **end-to-end delay**, D, is defined as the number of time slots it takes for the packet at the head of the source node to successfully hop to the destination.

r-TDMA-based Line Network: Steady State

- We primarily study the system behavior in the long-time limit.
- In the long time limit ($t \to \infty$), the probability of finding the system in configuration $\boldsymbol{\tau}[t] = (\tau_1[t], \ldots, \tau_N[t])$ becomes independent of t.
- With $\alpha = \beta = 1$ and $p = p_s$,

$$P(\boldsymbol{\tau}) = \frac{\langle W| \prod_{i=1}^{N} (\tau_i D + (1-\tau_i)E)|V\rangle}{\langle W|C^N|V\rangle}, \quad (\text{“bra-ket” notation})$$

where

$$D = \frac{1}{p_s}\begin{pmatrix} 1 & 1 & 0 & 0 & \ldots \\ 0 & 1 & 1 & 0 & \ldots \\ 0 & 0 & 1 & 1 & \ldots \\ 0 & 0 & 0 & 1 & \ldots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}, \quad E = D^T, \quad C = D + E$$

$$\langle W| = (1, 0, 0, \ldots) \quad \text{and} \quad |V\rangle = (1, 0, 0, \ldots)^T.$$

Steady State Probabilities

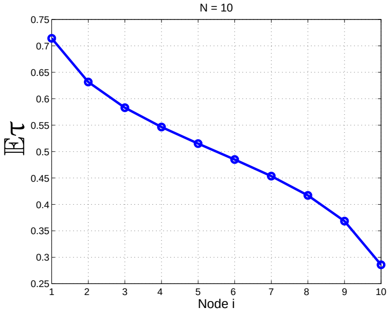
- For small N, the steady state probabilities may be computed in a straightforward manner.
  - For e.g., when $N = 1$,
    $P(0) = \frac{\langle W|E|V\rangle}{\langle W|C|V\rangle} = 1/2$, and $P(1) = \frac{\langle W|D|V\rangle}{\langle W|C|V\rangle} = 1/2$.
- For large N, we may use the following properties to compute the P's.
  - $C = D + E = p_s DE$.
  - $p_s^N \langle W|C^N|V\rangle = \frac{(2N+2)!}{(N+2)!(N+1)!}$.
  - For e.g., when $N = 6$,
    $P(1, 0, 1, 0, 1, 0) = 0.0326$.

r-TDMA-based Line Network: Steady State Occupancies

- The occupancy of node i, $1 \leqslant i \leqslant N$

$$\mathbb{P}(\tau_i = 1) = \mathbb{E}\tau_i = \frac{1}{2} + \frac{1}{4}\frac{(2i)!}{(i!)^2}\frac{(N!)^2}{(2N+1)!}\frac{(2N-2i+2)!}{[(N-i+1)!]^2}(N-2i+1).$$

- **The occupancies are independent of p_s!**



Notice the particle-hole symmetry. $\mathbb{E}\tau_i = 1 - \mathbb{E}\tau_{N+1-i}$.

r-TDMA-based Line Network: Steady State Throughput

Theorem

For the r-TDMA-based line network with N *nodes, the throughput (or "current") at steady state is*

$$T = \frac{p_s \mathbb{E}\tau_N}{N+1} = \frac{p_s(N+2)}{2(N+1)(2N+1)}.$$

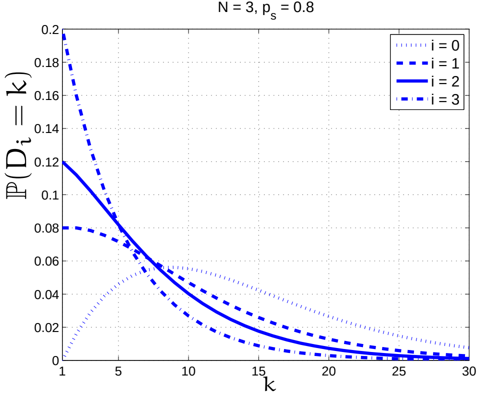
- $T \leqslant p_s/4$ and $T \sim p_s/(4N)$.
- Since the network reliability is 100%, the rate of packet flow is a constant, i.e., $T = p_s \mathbb{E}[\tau_i(1-\tau_{i+1})]/(N+1)$.

Corollary

(Little's Theorem) The average in-network end-to-end delay is

$$\mathbb{E}D_{e2e} = \frac{\sum_{i=0}^{N} \mathbb{E}\tau_i}{T} = \frac{2N^2+3N+1}{p_s}.$$

We have also derived the pmf of the packet delay at node i, $0 \leqslant i \leqslant N$.

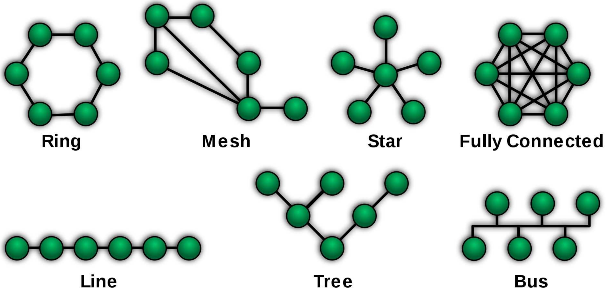


Only the delay at the final relay node follows a geometric distribution.

Remarks

- Our analysis is clean, yet rigorous, and thus allows for a good understanding of the dynamics of packet transport in line networks.
- Our results are scalable with the number of nodes, and thus offer insights towards solving issues such as the *long-hop versus the short-hop* problem in line networks.
- The TASEP is a powerful tool for the modeling and analysis of line networks.

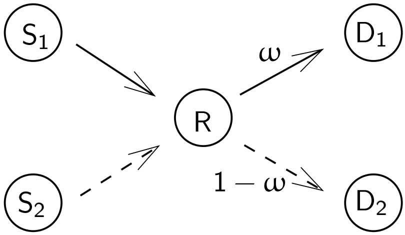
Part 2: Throughput Analysis of Networks with Complex Topologies



Introduction

- In general, ad hoc networks have complex topologies.
  - Multiple source-destination pairs.
  - Intersecting routes: merging and splitting of routes.
- Prior work uses *Kleinrock's independence assumption* to neglect correlations between flows.
- Propose the **partial mean-field approximation (PMFA)**, a framework that helps tightly approximate the throughput of networks with arbitrary topologies.

Two Two-hop Flows via a Common Relay



- Two flows $S_1 \rightarrow R \rightarrow D_1$ and $S_2 \rightarrow R \rightarrow D_2$ occurring via the common relay node R.
- R has a buffer size of 2, one for each flow passing through it.
- Priority-based scheduling with parameter ω.

Two Two-hop Flows via a Common Relay (Contd.)

- Let $\tau_j^{[i]}$ represent the steady state configurations for the buffers across the two flows, $i = \{1, 2\}$, for each of the three nodes involved in each flow, numbered $j = \{0, 1, 2\}$.
- The throughput across each link is the same:

$$\mathbb{E}\left[1 - \tau_1^{[1]}\right] = \mathbb{E}\left[\tau_1^{[1]}\left(1 - (1 - \omega)\,\tau_1^{[2]}\right)\right],$$

and

$$\mathbb{E}\left[1 - \tau_1^{[2]}\right] = \mathbb{E}\left[\tau_1^{[2]}\left(1 - \omega\tau_1^{[1]}\right)\right].$$

Two Two-hop Flows via a Common Relay (Contd.)

- Use the *mean-field approximation* (MFA), according to which all the correlations between the buffer occupancies are neglected.

$$\mathbb{E}\left[\tau_i^{[j]}\tau_k^{[l]}\right] = \mathbb{E}\tau_i^{[j]}\mathbb{E}\tau_k^{[l]},$$

- Solving the two equations simultaneously, we obtain

$$T^{[1]}(\omega) = \frac{p_s \mathbb{E}\tau_1^{[1]}}{3} = \frac{p_s\left(2\omega - 3 + \sqrt{4\omega^2 - 4\omega + 9}\right)}{12\omega}.$$

- $T^{[1]}(1) = p_s/6$, while $T^{[1]}(0) = p_s/9$.
  $T^{[1]}(0.5) = T^{[2]}(0.5) = p_s(\sqrt{2} - 1)/3$.

The Partial MFA (PMFA)

Theorem

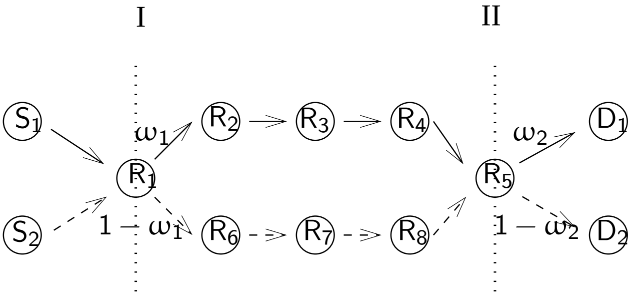
The throughput across a cut in the network comprising n *nodes with influx and outflux rates and hopping probability* α*,* β *and* p_s *respectively is given by*

$$T(\alpha, \beta, n) = \begin{cases} p_s/(N+1) \times \min\{\alpha, \beta\} & n = 0 \\ p_s/(N+1) \times \frac{Z(\alpha,\beta,n-1)}{Z(\alpha,\beta,n)} & n \geqslant 1, \end{cases}$$

where $Z(\alpha, \beta, 0) = 1$ *and*

$$Z(\alpha, \beta, n) = \sum_{i=1}^{n} \frac{i(2n-1-i)!}{n!(n-i)!} \frac{(1/\beta)^{i+1} - (1/\alpha)^{i+1}}{1/\beta - 1/\alpha}, n \geqslant 1.$$

A Toy Example



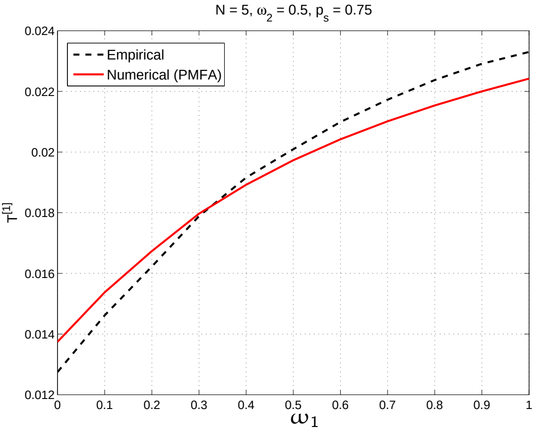
A toy example consisting of two multihop flows $S_1 \rightarrow D_1$ and $S_2 \rightarrow D_2$. The dotted lines I and II represent two cuts along the flow.

Set $\mathbb{E}\tau_1^{[1]} = x$, $\mathbb{E}\tau_5^{[1]} = y$, $\mathbb{E}\tau_1^{[2]} = z$ and $\mathbb{E}\tau_5^{[2]} = w$.

$$T(1,1-x,1) = T(x(1-(1-\omega_1)z), 1-y, 3), \qquad T(1,1-x,1) = T(y(1-(1-\omega_2)w),1,1)$$

$$T(1,1-z,1) = T(z(1-\omega_1 x), 1-w, 3), \qquad T(1,1-z,1) = T(w(1-\omega_2 y),1,1)$$

A Toy Example (Contd.)



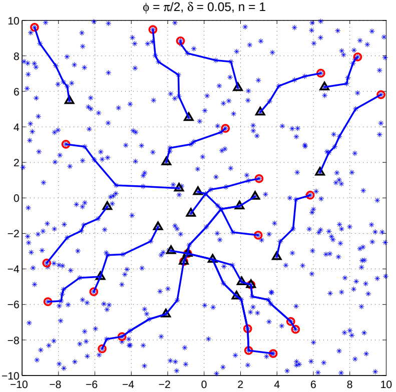
Part 3: TDR Tradeoffs in Multihop Networks with Random Access

Introduction

- Performance goals in multihop networks often conflict with one another.
  - Hardly possible to guarantee a high rate of transmission in conjunction with reliable packet delivery and low latency.
- In scenarios where reliable delivery is not critical, one can have the nodes forcibly drop a small fraction of packets.
- We characterize the throughput-delay-reliability (TDR) tradeoffs in multihop networks.

System Model

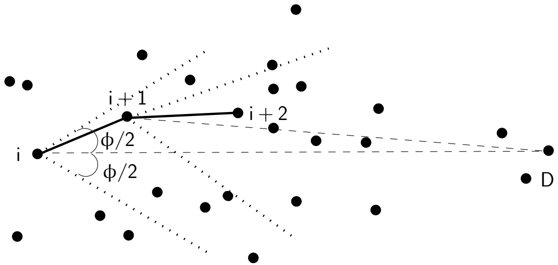
- **Source nodes**: homogeneous PPP (δ).
- **Relays and destinations**: homogeneous PPP ($1 - \delta$).
- For each source node, the destination node is chosen at a random orientation, and at a random finite distance.
- Homogeneous network => sufficient to consider the "typical" flow.



Each destination is assumed to be located 5 nearest-neighbor ($n = 1$) hops away from its source.

System Model (Contd.)

- **Routing**: each node that receives a packet relays it to its n^{th}-nearest-neighbor ($n \geqslant 1$) in a sector of angle $\phi \in [0, \pi]$ towards the destination.



MAC Scheme: **Slotted ALOHA**

- In each time slot, every node *having a packet* independently transmits w.p. q or remains idle w.p. $1 - q$.

$=>$ simultaneous transmissions lead to interference.

Performance Metrics: **TDR**

- The per-flow **throughput** T, is defined as the average number of packets successfully delivered (to the destination) in unit time, along a typical flow in the network.
- The **mean end-to-end delay**, D, is defined as the average number of time slots it takes for the packet at the head of the source node to successfully hop to the destination.
- The **end-to-end reliability** R is defined as the fraction of packets generated at the source that are eventually delivered.

The Regime $R = 1$

Use some ideas from the *parallel* TASEP literature.

Theorem

For an ALOHA-based line flow along N *relays, the steady state throughput at full reliability (*$R = 1$*) is*

$$T = \frac{qp_s B(N)}{B(N+1) + qp_s B(N)},$$

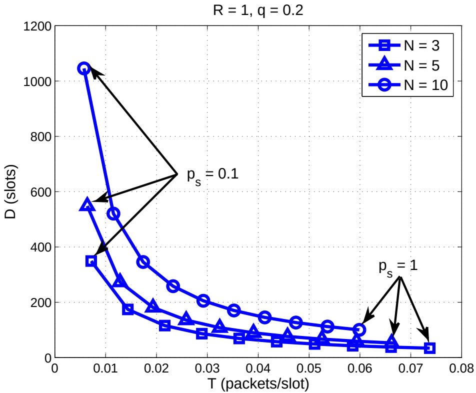
while the average end-to-end delay is given by

$$D = (1 + N/2)/T.$$

where $B(0) = 1$, and

$$B(k) = \sum_{j=0}^{k-1} \frac{1}{k} \binom{k}{j} \binom{k}{j+1} (1 - qp_s)^j, \quad k > 0.$$

The Regime $R = 1$ (Contd.)



For each value of N, the TD curve is a hyperbola.

The Regime $R < 1$

- When $R = 1$, D and T performances are poor at small p_s.
- Nodes can choose to drop a small fraction of packets ($R < 1$).
  - Each node having a packet decides to drop the packet in its buffer or not stochastically w.p. ξ.

For clarity, we consider the following two regimes separately.

The Noise-Limited Regime: $p_s = \Pr(\mathrm{SNR} > \Theta)$

Noise power in the network is much stronger than the interference.

The Interference-Limited Regime: $p_s = \Pr(\mathrm{SIR} > \Theta)$

- Interference power in the network is much stronger than noise.
- Also covers the regime wherein the interference and noise powers are comparable.

The Regime $R < 1$ (Contd.)

- System Evolution: The following events affect τ_i:
  a) Node $i-1$ transmits its packet to node i.
  b) Node i transmits its packet to node $i+1$.
  c) Node i drops its packet.
- Employing mean-field theory, we obtain at steady state, $\mathbb{E} \lim_{t\to\infty} \Delta\tau_i[t] = 0$ for $1 \leqslant i \leqslant N$, i.e.,

$$p_s(1-\xi)q\Big[\underbrace{\mathbb{E}\tau_{i-1}(1-\mathbb{E}\tau_i)}_{a)} - \underbrace{\mathbb{E}\tau_i(1-\mathbb{E}\tau_{i+1})}_{b)}\Big] - \underbrace{\xi\mathbb{E}\tau_i}_{c)} = 0.$$

- The steady state occupancies $\mathbb{E}\tau_i$ may be obtained by numerically solving these N non-linear equations.

The Regime $R < 1$ (Contd.)

1. The steady-state throughput is

$$T = qp_s \mathbb{E}\tau_N.$$

2. The mean end-to-end delay is

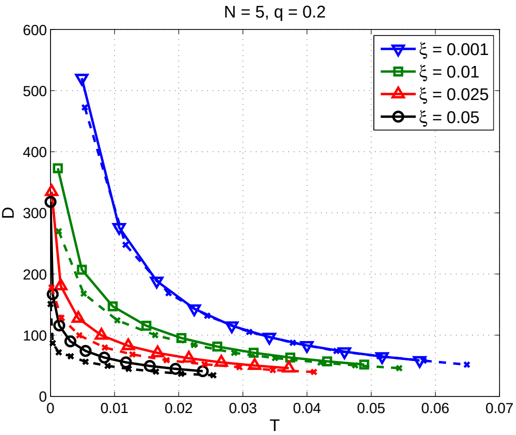
$$D = \sum_{i=0}^{N} s_i^{-1},$$

where $s_i = qp_s(1 - \mathbb{E}\tau_{i+1})$.

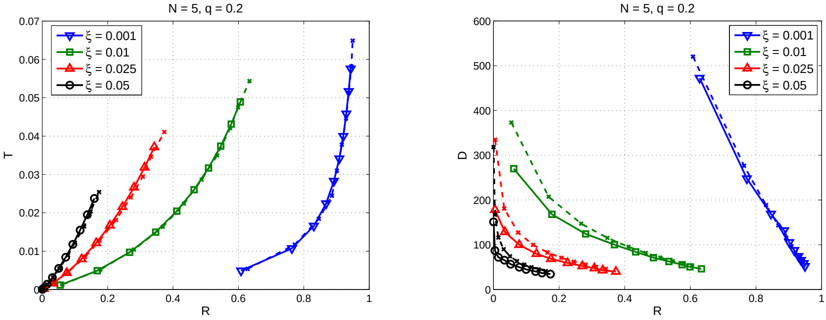
3. The end-to-end reliability of the network is

$$R = \prod_{i=0}^{N} \frac{s_i(1-\xi)}{s_i + \xi - s_i\xi}.$$

The Noise-Limited Regime; $R < 1$



Increasing ξ helps reduce the end-to-end delay significantly



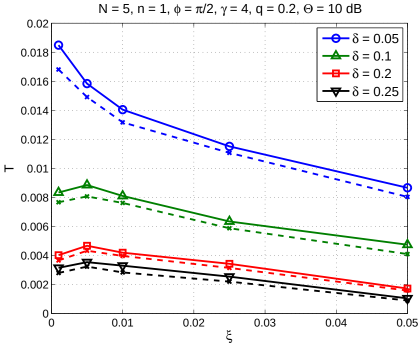
However, the throughput and reliability performances worsen.

The Interference-Limited Regime; $R < 1$

- The average number of potential interferers in each flow is $1 + \sum_{i=1}^{N} \mathbb{E}\tau_i$.
  - The set of interferers (approximately) forms a PPP with density $\lambda_I = \delta q \left(1 + \sum_{i=1}^{N} \mathbb{E}\tau_i\right)$.
- The probability of a successful transmission for a typical link is

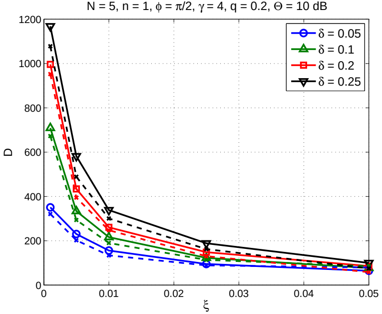
$$p_s \gtrapprox \left(\frac{(1-\delta)\phi}{(1-\delta)\phi + 2\delta q \left(1 + \sum_{i=1}^{N} \mathbb{E}\tau_i\right) c} \right)^n .$$

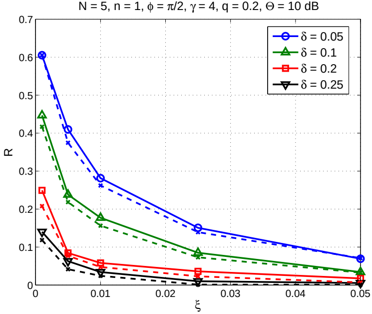
The Interference-Limited Regime; $R < 1$ (Contd.)



- When δ is small, increasing the packet dropping probability ξ reduces the system throughput.
- As δ gets larger, dropping a few packets helps mitigate the interference, and the throughput across a typical flow improves.

The Interference-Limited Regime; $R < 1$ (Contd.)





With increasing ξ or decreasing δ, the mean end-to-end delay decreases; the reliability also suffers.

Summary

- TASEP is a strong tool for analyzing multihop networks.
- We hope that this introductory work instigates interest in solving other relevant wireless networking problems:
  - Other MAC schemes: ordered sequential (TDMA), sublattice parallel update (spatial TDMA).
  - Bidirectional traffic: ASEP.
  - Multiple-sized buffers: K-exclusion processes.
  - TDR tradeoffs for other traffic models and sophisticated packet dropping strategies: Langmuir kinetics

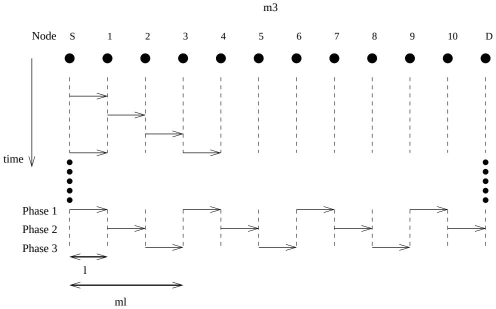
Relevant References

- S. Srinivasa and M. Haenggi, "The TASEP: a statistical mechanics tool to study the performance of wireless line networks," *IEEE International Conference on Computer Communication Networks*, Aug. 2010.
- S. Srinivasa and M. Haenggi, "A statistical mechanics-based framework to analyze ad hoc networks with random access," *IEEE Transactions on Mobile Computing*, 2011. Accepted.
- S. Srinivasa and M. Haenggi, "Combining stochastic geometry and statistical mechanics for the analysis and design of mesh networks," *Elsevier Ad Hoc Networks*, 2011. Accepted.
- S. Srinivasa and M. Haenggi, "Throughput-delay-reliability tradeoffs in multihop networks with random access," *Allerton Conference on Communication, Control and Computing*, Sep. 2010.

All papers are available at `www.nd.edu/~mhaenggi/pubs/`.

EXTRA SLIDES.

Optimality of ALOHA: The Sufficiency of Small Buffers



The optimal scheduling assignment for a line network with $N = 10$.

- Optimal spatial reuse parameter is $m = 3$.
- Interference induces a natural spacing between packets.
- ALOHA with contention probability 1 is optimal.
- Small buffers (as low as unit size) are sufficient for the optimal operation of the network.

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