Delay and Throughput Correlations in Ad Hoc Networks

Sunil Srinivasa and Martin Haenggi

IT MANET PI Meeting, Austin, May 24-26 2010.

UNIVERSITY OF NOTRE DAME

Overview

- Draw analogies between flows in an ad hoc network and the TASEP.
- Characterize the correlations between delays in a line network.
- Study the throughput performance of networks with intersecting routes.

Limitations in Prior work:

- Kleinrock independence approximation.
- Focus on small networks [Ryoki 02, Daduna 00].
- Backlogged nodes, simplistic assumptions [Conti 00].

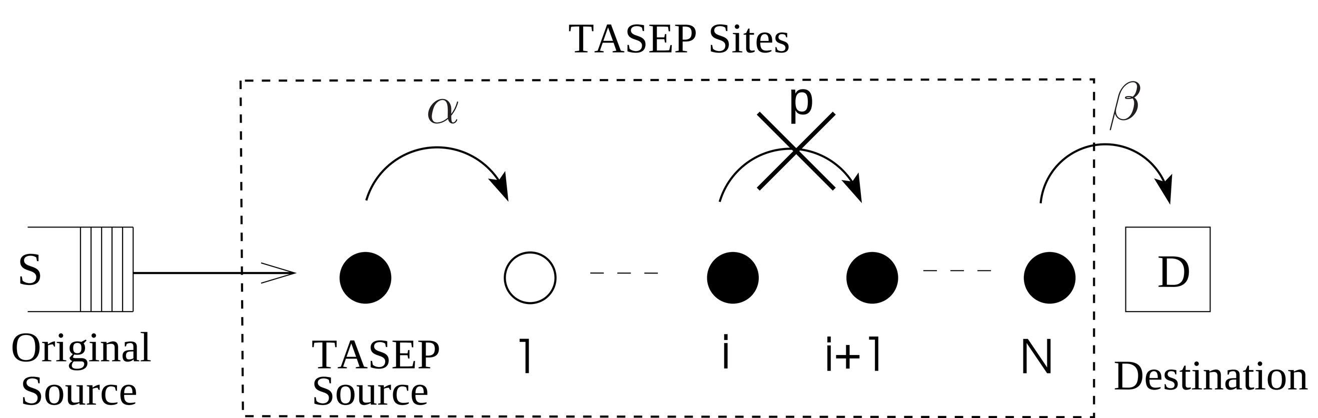
System Model

- Several flows, each occurring over an infinite duration of time.
- *Randomized TDMA (r-TDMA)*: In each time slot, the transmitting node is chosen uniformly randomly from the set of all nodes in the network.
- Link reliability: $p_s = \mathbb{P}(\text{SINR} \geq \Theta)$.

The Totally Asymmetric Simple Exclusion Process

A Revised Transmission Policy:

1. All the buffering in the network is performed at source nodes.
2. Transmissions are not attempted if the adjacent relay's buffer already contains a packet.
3. Packets are retransmitted until successfully received.



TASEP equivalence of a network flow: A successful transmission is possible only when $\{\tau_i, \tau_{i+1}\} = \{1, 0\}$.

The Matrix Product Ansatz

The steady state probabilities of finding the system in the configuration $\{\tau\} = \{\tau_1, \tau_2, \ldots, \tau_N\}$, $\tau_i \in \{0,1\}$ is

$$\mathbb{P}^{(N)}(\tau) = \frac{\langle W | \prod_{i=1}^{N} (\tau_i D + (1-\tau_i)E) | V \rangle}{\langle W | C^N | V \rangle},$$

where

$$D = \frac{1}{p}\begin{pmatrix} p/\beta & \gamma_1 & 0 & \cdots \\ 0 & 1 & 1 & \cdots \\ 0 & 0 & 1 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}, E = \frac{1}{p}\begin{pmatrix} p(1-\alpha)/\alpha & 0 & 0 & \cdots \\ \gamma_2 & 1-p & 0 & \cdots \\ 0 & 1-p & 1-p & \cdots \\ 0 & 0 & 1-p & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix},$$

$$\langle W | = (1, 0, 0, \ldots) \quad \text{and} \quad |V\rangle = (1, 0, 0, \ldots)^T.$$

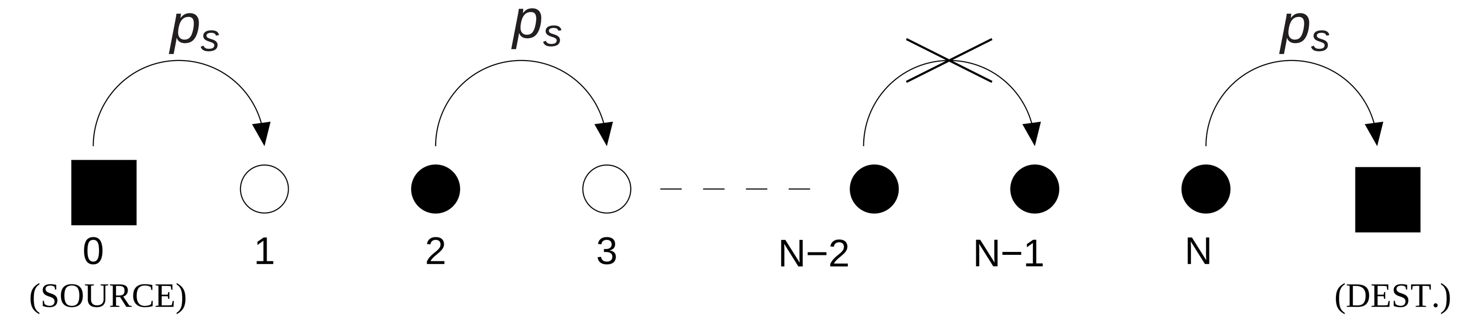
Here, γ_1 and γ_2 are chosen so as to satisfy

$$\gamma_1\gamma_2 = \frac{p}{\alpha\beta}[1 - p - (1-\alpha)(1-\beta)].$$

Some useful properties:

- $D + E = pDE$.
- $\langle W | C^N | V \rangle := \eta(N) = \frac{(2N+2)!}{(N+2)!(N+1)!}$.

Delay Correlations in Line Networks

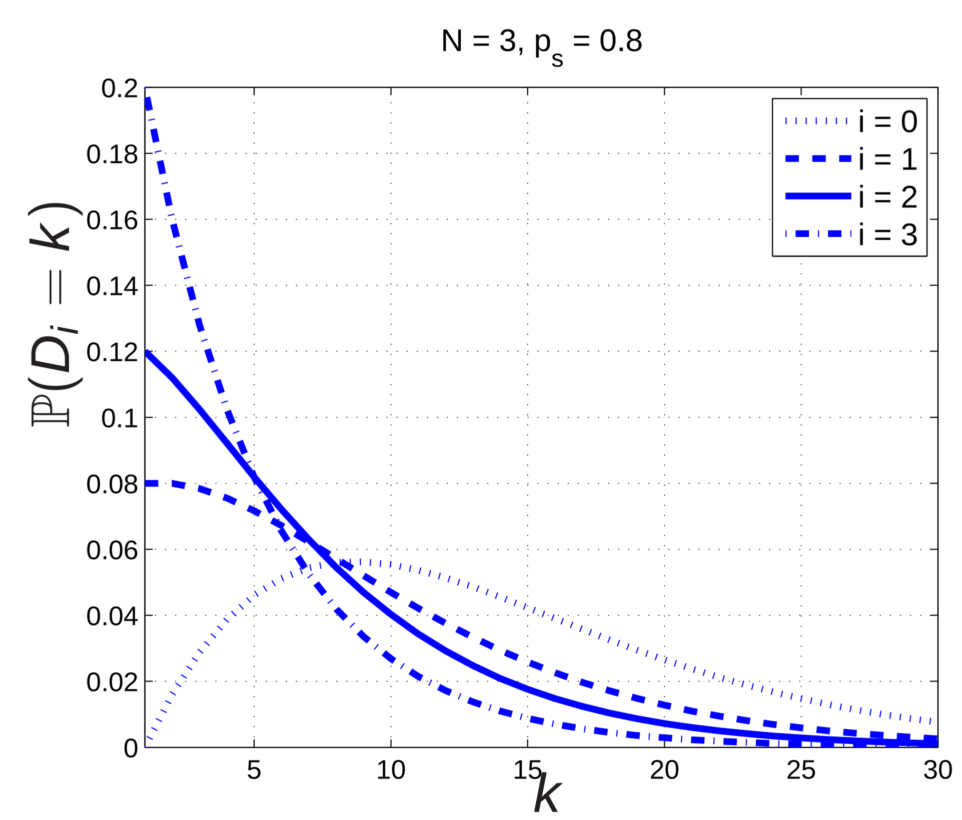


Theorem [Delay pdf] The pdf of the packet delay at node i, $0 \leq i \leq N$ is given by

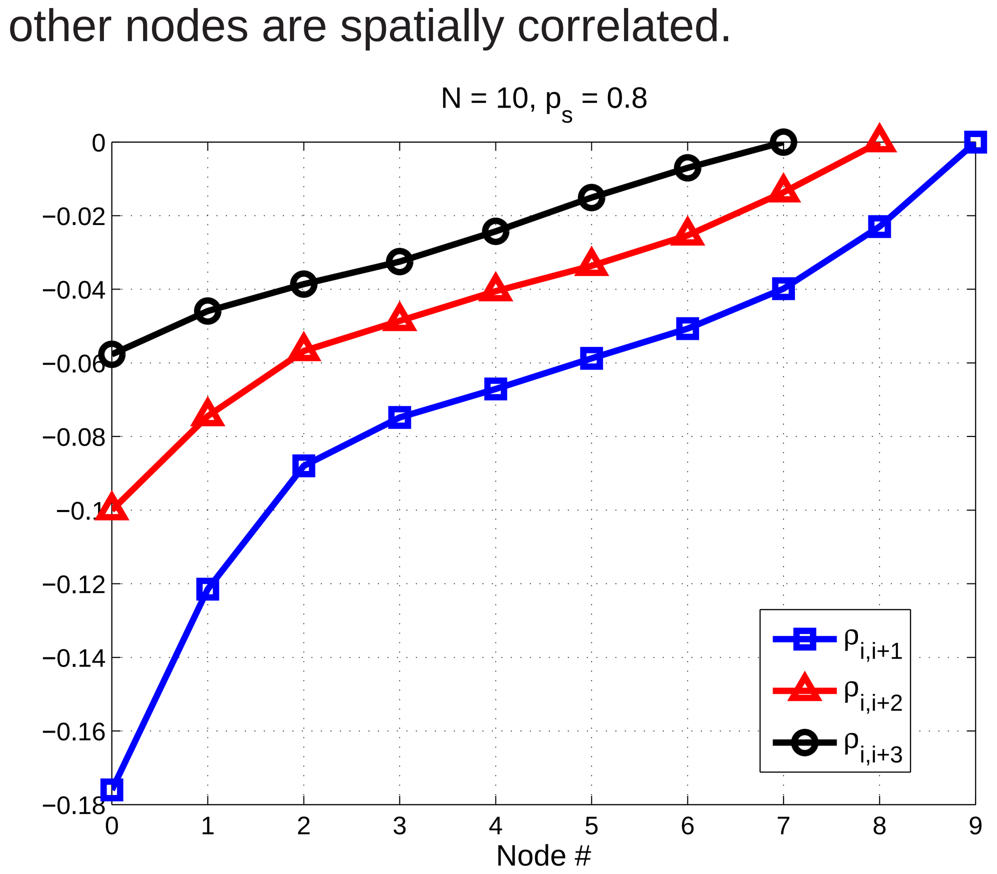
$$\mathbb{P}(D_i = k) = \sum_{j=0}^{N-i} \Delta_{i,j}^{(N)} \binom{k-1}{j} \xi^{j+1}(1-\xi)^{k-1-j},$$

where $\xi = p_s/(N+1)$, and

$$\Delta_{i,j}^{(N)} = \sum_{k=0}^{\lfloor \frac{i-1}{2} \rfloor} (-1)^k \frac{\eta(N-k-2)}{\eta(N-1)} \times \left[\binom{j-k-2}{k} \mathbb{E}\tau_i^{(N-k-2)} + \binom{j-k-2}{k-1}\right].$$



- Delay at final relay node follows a geometric distribution and is independent of other delays.
- Delays at other nodes are spatially correlated.



- Spatial Delay correlation coefficients are negative.
- Correlations between delays at nodes closer to the source are higher.
- Correlations between delays at nodes farther apart are smaller.

Networks Comprising Intersecting Routes

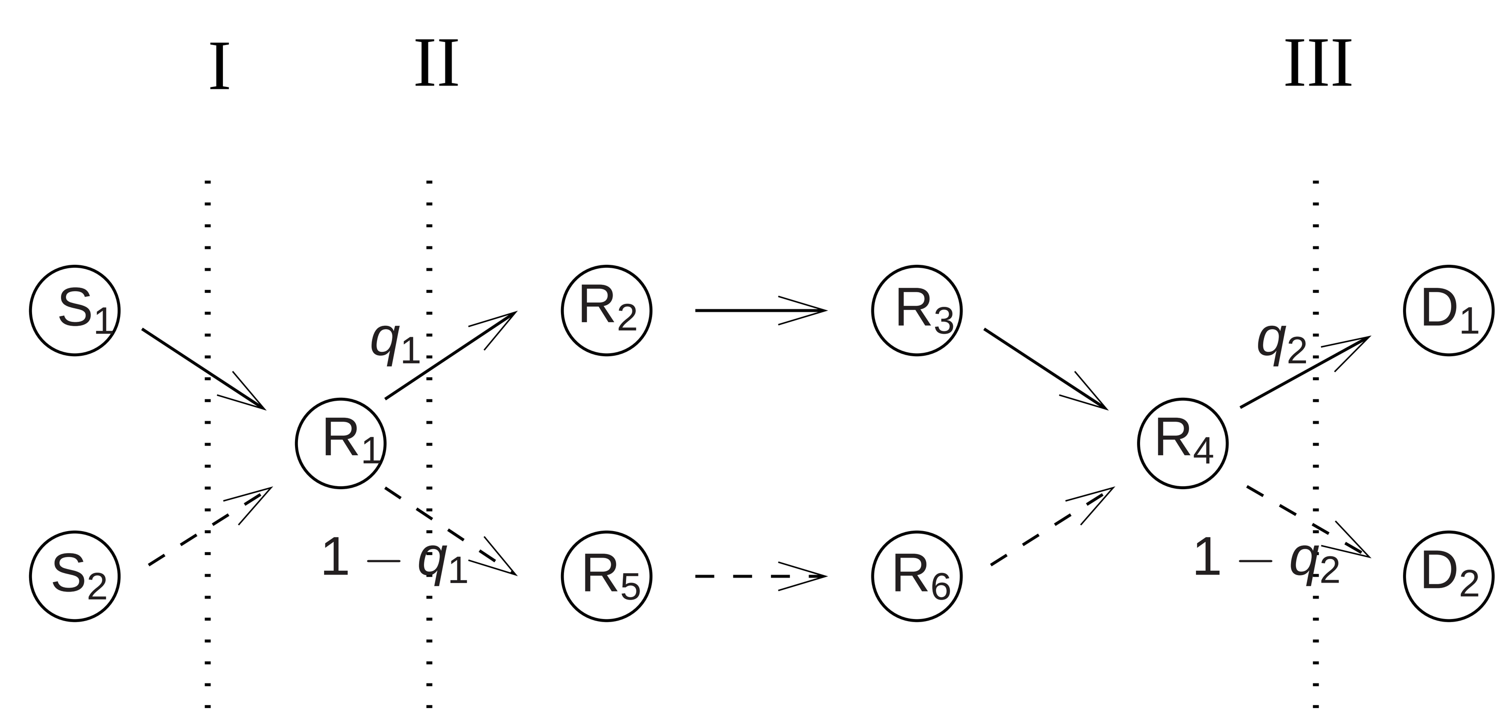
Theorem The throughput across a network flow over n relay nodes with influx rate α, outflux rate β and hopping probability p_s is

$$T(\alpha, \beta, p_s, 0) = \begin{cases} p_s \min\{\alpha, \beta\} & n = 0 \\ p_s\alpha\beta/(2\alpha + 2\beta) & n = 1 \\ \xi p_s Z(\alpha, \beta, n-1)/Z(\alpha, \beta, n) & n \geq 2 \end{cases}$$

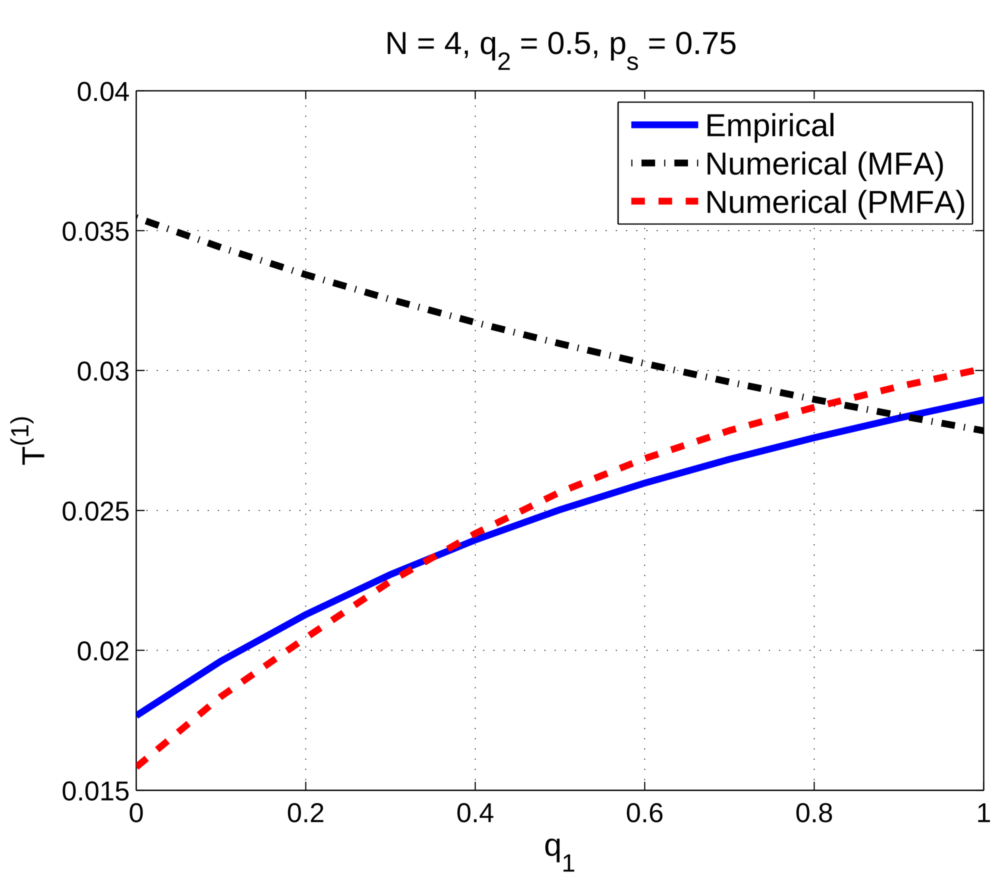
where $\xi = 1/(n+1)$, and

$$Z(\alpha, \beta, n) = \sum_{i=i}^{n} \frac{i(2n-1-i)!}{n!(n-i)!} \frac{(1/\beta)^{i+1} - (1/\alpha)^{i+1}}{1/\beta - 1/\alpha}.$$

A Toy Example



i) I, II and III represent the three cuts along the network flow.
ii) The rate of packet flow, T, across each cut is conserved.
iii) Neglect correlations between node occupancies at common relays.
iv) Numerically evaluate the throughput across each flow.



The partial mean field approximation (PMFA) method that we employ is much more accurate than the MFA.

Future Work

- Design retransmission algorithms based on network correlations.
- Analyze ad hoc networks with relays serving ≥ 2 flows.
- Characterize the performance of systems running other MAC schemes, in particular, slotted ALOHA, CSMA and spatial TDMA.