The TASEP: A Statistical Mechanics Tool to Study the Performance of Wireless Line Networks

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Multi-hop Wireless Networks

- Due to the stringent energy constraint in nodes and interference, a natural communication strategy is to reduce range of transmission.
- Multi-hop networks are not just meant to carry small volumes of data, but may also be intended for broadband services, e.g. mesh networks.
- However, existing buffering policies have inherent drawbacks: large queueing delays, non-coordinated transmissions, buffer overflows [Fu '03], [Xu '01].
- Consequently, the end-to-end delay and throughput performance in such systems is disappointing.

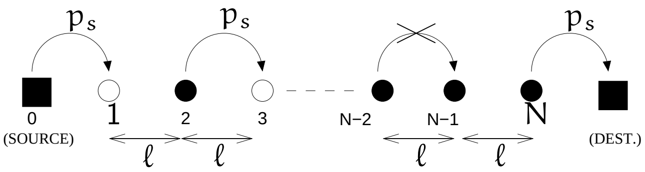
Prior Work:

- Queueing-theory based; less tractable [Xie '09], [Bisnik '09].
- Considered small [Ryoki '02] or infinite networks [Gupta '00].
- Neglected queueing delays [Yang '03].
- Assumed all nodes to be backlogged [Abouei '07].

Our Contributions:

- Propose a revised buffering scheme for multihop networks.
- Draw analogies between the **totally asymmetric simple exclusion process** (TASEP) and wireless line networks.
- Tap into the rich theory of TASEP and its results to
  - Analyze steady state end-to-end delay and throughput.
  - Provide insights into network design.

System Model



- All nodes use the same channel.
- Attenuation in the channel: modeled as the product of
  - Large-scale path loss with exponent γ.
  - Small-scale Rayleigh block fading.
- Transmission success events are dictated by the SINR model.

$$p_s = \mathbb{P}(\mathsf{SINR} > \Theta),$$

p_s: Success probability across each link.
Θ: SINR threshold.

A Revised Buffering and Transmission Policy

1. All the buffering is pushed back to the source, while relay nodes have buffer sizes of unity.
   Furthermore, the source node is always backlogged.
2. Nodes do not accept incoming packets if their buffer is already full.
   - Simple way to prevent packets from getting too close.
   - Self-organization:
     - Transmitting nodes are at least two hops apart.
     - The exclusion principe regulates the traffic injected in a backpressure-like manner.
3. Packets are retransmitted until they are successfully received. (100% network reliability).

Advantages of the Single-Buffer Scheme

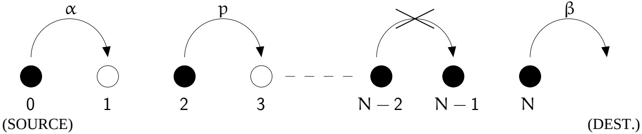
- Lowers average in-network delay.
  - Stacking-up of packets in buffers is minimal.
- Lessens the variance of the delay.
  - Packet delays are more tightly controlled.
  - Depending on the time a packet spends in its buffer, the source itself can judiciously decide whether to drop it or not.
- Reduces hardware cost and energy consumption.
- Minimizes end-to-end buffer usage [Venkataramanan '10], provides buffering gain [Bhadra '06], self-organizes network operation [Dousse '07].

Totally Asymmetric Simple Exclusion Process (TASEP) with Open Boundaries

- A topic in statistical mechanics.
- Describes the dynamics of self-driven systems with several interacting particles.
- Applied in problems such as
  - Traffic flow modeling.
  - Kinetics of bipolymerization.
  - Stock market fluctuations.
- A paradigm for non-equilibrium systems.

TASEP Model

- The source site is numbered 0 and there are N other sites.
- **Configuration** of the sites : $\tau_i[t] \in \{0, 1\}$ - occupied or not.
- Hopping between sites at time t is possible only if the configuration $\{\tau_i[t], \tau_{i+1}[t]\}$ is $\{1,0\}$.



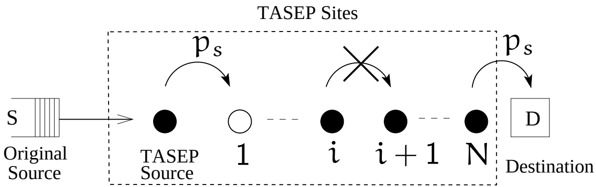
Snapshot of the TASEP system model. Filled circles indicate **occupied sites** and the rest indicate **holes**.

- Exclusion principle creates a **particle-hole duality**.

TASEP ≡ Wireless Line Networks?

Note the analogies:

- Sites ⇔ Nodes.
- Particles ⇔ Packets.
- Exclusion principle ⇔ Unit buffer sizes.
- Hopping probability ⇔ Link reliability.



The wireless line network is modeled as a source node with a large buffer connected to the TASEP particle flow model.

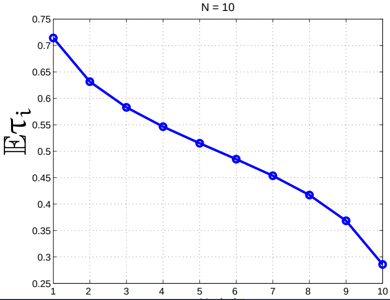
MAC Schemes

1. **Slotted ALOHA**: In each time slot, each node having a packet independently transmits with a probability of contention q.
2. **Randomized-TDMA (r-TDMA)**: The transmitting node in each time slot is chosen uniformly randomly from the set of all nodes (with probability $1/(N+1)$) instead of being picked in an ordered fashion.
    - CSMA-type scheme (at most one transmitter in each slot).
    - Limiting form of ALOHA ($q \to 0$).

Analysis of the r-TDMA-based Network

- $\mathbb{P}(\tau_i[t] = 1)$: **occupancy** of node i's buffer, in time slot t.
- We are interested in steady state performance ($t \to \infty$).
- $\tau_i :\triangleq \lim_{t\to\infty} \tau_i[t]$; $\mathbb{P}(\tau_i = 1) = \mathbb{E}\tau_i$.
- For $0 \leqslant i \leqslant N$,

$$\mathbb{E}\tau_i = \frac{1}{2} + \frac{1}{4}\frac{(2i)!}{(i!)^2}\frac{(N!)^2}{(2N+1)!}\frac{(2N-2i+2)!}{[(N-i+1)!]^2}(N-2i+1).$$



- The occupancies are independent of p_s!
- Particle-hole symmetry ($\mathbb{E}\tau_i = 1 - \mathbb{E}\tau_{N+1-i}$).

Analysis of the r-TDMA-based Network (Contd.)

The throughput at steady state is

$$T = \frac{p_s \mathbb{E}\tau_N}{N+1} = \frac{p_s(N+2)}{2(N+1)(2N+1)}.$$

- T is upper-bounded by $p_s/4$.
- T decreases with increasing system size $(T \sim p_s/(4N))$.

The average end-to-end delay is

$$\mathbb{E}D_{\mathrm{e2e}} = \frac{(2N^2+3N+1)}{p_s}.$$

- Consequence of Little's theorem.
- $\mathbb{E}D_{\mathrm{e2e}}$ grows quadratically with N.

Short-hop Versus Long-hop Routing

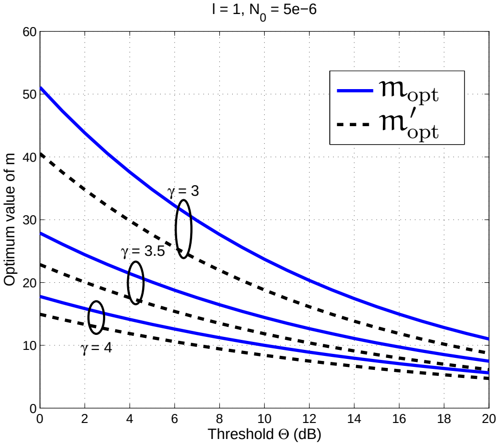
- **Question**: Is it beneficial to route over many short hops or a smaller number of longer hops?
- Suppose that communication occurs across nodes that are m hops apart.
- Delay-minimizing hopping parameter is

$$m_{\text{opt}} = \frac{1}{\ell}\left(\frac{2}{\Theta N_0 \gamma}\right)^{1/\gamma}.$$

- Throughput-maximizing hopping parameter is

$$m'_{\text{opt}} = \frac{1}{\ell}\left(\frac{1}{\Theta N_0 \gamma}\right)^{1/\gamma}.$$

Short-hop Versus Long-hop Routing (Contd.)



$$m_{opt} = m'_{opt} 2^{1/\gamma}$$

Analysis of the ALOHA-based Network

- The "effective" hopping probability is $p = qp_s$.
- The steady state occupancies are given by

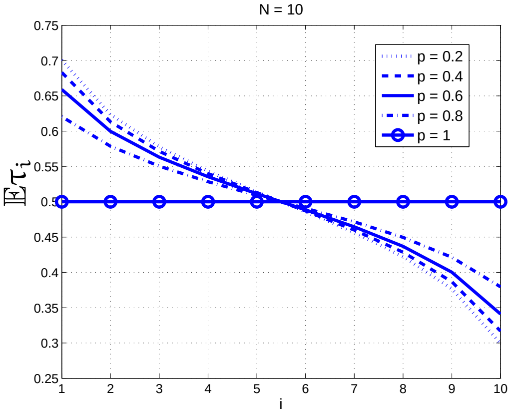
$$\mathbb{E}\tau_i = \frac{(1 - qp_s)\sum_{n=0}^{N-i} B(N-n)B(n) + qp_s B(N)}{B(N+1) + qp_s B(N)},$$

where $B(0) = 1$, and

$$B(k) = \sum_{j=0}^{k-1} \frac{1}{k}\binom{k}{j}\binom{k}{j+1}(1 - qp_s)^j, \quad k > 0.$$

- When $N \gg 1$, $\mathbb{E}\tau_1 = \left(2p - 1 + \sqrt{1-p}\right)/2p$ and $\mathbb{E}\tau_N = \left(1 - \sqrt{1-p}\right)/2p$. Also, $\mathbb{E}\tau_i \approx 1/2$ for $1 < i < N$.

Analysis of the ALOHA-based Network (Contd.)



Unlike the r-TDMA case, $\mathbb{E}\tau_i$ critically depends on p.

Analysis of the ALOHA-based Network (Contd.)

The steady state throughput is

$$T = qp_s \mathbb{E}\tau_N = \frac{qp_s B(N)}{B(N+1) + qp_s B(N)}.$$

- For $N \gg 1$, the network throughput at steady state is

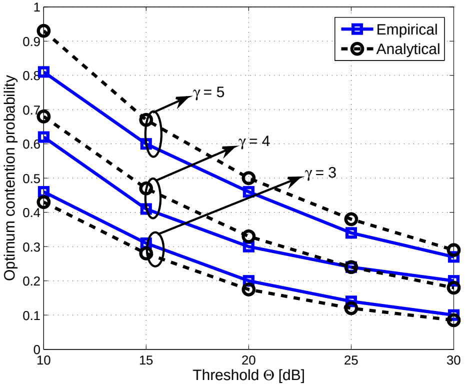
$$T \sim \left(1 - \sqrt{1 - qp_s}\right) \Big/ 2.$$

- From Little's theorem, $\mathbb{E}D_{\mathrm{e2e}} = (1 + N/2)/T$.
- For the special case $q = p_s = 1$, every alternate node transmits successfully in each time slot; $T = 1/2$, and $\mathbb{E}D_{\mathrm{e2e}} = 2$.

Optimizing the Contention Probability

- A long ($N \gg 1$) ALOHA-based line network.
- Consider the interference-limited regime ($p_s = \mathbb{P}(\mathrm{SIR} > \Theta)$).
- **Question**: What value of q minimizes the end-to-end delay?
  - Small q: nodes hold on to packets for long.
  - Large q: interference in the network is high.
- **Result**: the optimum value of q is $q_{\mathrm{opt}} = \min\{1, 2/c\}$, where $c = \pi\Theta^{1/\gamma} \Big/ \sqrt{\gamma/2} - 1$.
- The same q maximizes the network throughput as well.

Optimizing the Contention Probability (Contd.)



For small Θ, (almost) all nodes having a packet can transmit; interference induces a natural spacing between transmitting nodes.

Summary

- Proposed a transmission policy for multihop networks that helps regulate the flow of packets in a completely decentralized manner.
- Used some ideas from TASEP to characterize the steady state end-to-end delay and throughput performances of wireless line networks.
- Obtained results that are scalable with the number of nodes, and thus can provide helpful insights into the design of ad hoc networks.
- Hope that this introductory work instigates interest in solving other relevant wireless networking problems employing ideas from statistical mechanics.