The TASEP: A Statistical Mechanics Tool to Study the Performance of Wireless Line Networks

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***Abstract*—We consider a multihop wireless line network with a single unidirectional data flow and show that by limiting the buffer sizes at the relay nodes to unity, the flow of traffic in the system can be efficiently regulated in a completely distributed fashion. Upon exerting this simple transmission policy, we find that the transport of packets in the wireless network is analogous to the flow of particles in the totally asymmetric simple exclusion process (TASEP). Using existing results from statistical mechanics, we characterize the end-to-end delay and throughput performance of multihop wireless line networks for two different channel access schemes. Additionally, we apply our findings towards the design of long networks. This paper also aims at promoting the TASEP as a powerful tool for analyzing the performance of ad hoc networks.**

I. Introduction

A wireless ad hoc network is typically formed by deploying nodes that possess self-organizing capabilities. Due to the stringent energy constraint in these devices, a natural communication strategy to conserve battery life is to reduce the range of transmission and employ multihop routing, wherein relays assist in the delivery of packets from the source to the destination. Multihop wireless networks are not just intended to carry small volumes of data in an energy-efficient manner, but may also be used to provide broadband services under QoS constraints, for example in mesh networks. However, existing buffering schemes for multihop wireless networks involving large buffer sizes and a drop-tail policy have certain inherent drawbacks such as buffer overflows, excessive queueing delays and scheduling issues, resulting in uncoordinated transmissions. Consequently, the end-to-end delay and throughput performance in such systems is disappointing [1].

In this paper, we introduce a simple transmission policy that helps overcome the aforementioned shortcomings and analyze the delay and throughput performance of networks employing the policy. Our contribution is two-fold:
First, we propose a simple buffering scheme for multihop wireless networks, in which the buffer sizes at relay nodes are restricted to just one packet, and all the buffering is pushed back to the source node. We shall see that employing this modified transmission policy not only helps keep packet delays small but also helps regulate the flow of packets in a completely decentralized fashion.
Second, we characterize the end-to-end delay and achievable throughput of the wireless multihop line network employing the revised buffering policy for two different channel access schemes. Additionally, we employ our findings to provide useful design insights in long wireless line networks.

To simplify the analysis, we exploit the analogy between multihop wireless line networks and the discrete-time *totally asymmetric simple exclusion process* (TASEP) [2], a stochastic process in statistical mechanics, which has also been applied to the study of other interesting problems such as the kinetics of biopolymerization and traffic. This paper is intended to provide insight into the dynamics of packet transport in multihop wireless networks.

II. System Model

We consider a multihop wireless line network with a unidirectional data flow from the leftmost to the rightmost node. The source node S is numbered 0 and generates packets of fixed length at a constant rate. The network contains N relay nodes (numbered 1 through N) and a destination D, numbered $N+1$. The arrangement of nodes is regular (on a lattice) with a separation of l between any pair of adjacent nodes. Time is slotted to the duration of a packet, transmission attempts occur at slot boundaries, and each transmitting node transmits at unit power.

We assume that all the nodes in the network use the same channel; thus, simultaneous transmissions cause interference between links. We take the attenuation in the channel to be modeled as the product of a Rayleigh fading component and the large-scale path loss component with exponent γ. The noise in the network is taken to be AWGN with variance N_0. We define the transmission from node i to target node j to be successful if the (instantaneous) signal-to-interference-and-noise ratio (SINR) at j is greater than a predetermined threshold Θ. The probability of successful reception is denoted by $p_s = \Pr[\text{SINR} > \Theta]$.

A. A Revised Transmission Policy

We consider a simple transmission policy characterized by the following two rules.

1) All the buffering is performed at the source node, while relay nodes are essentially bufferless (i.e., have buffer sizes of unity). Furthermore, transmissions are not accepted by relay nodes if their buffer already contains a packet.

2) Packets are retransmitted until they are successfully received.

Using Rule 1 alone may lead to a loss in throughput due to dropped packets; Rule 2 is needed to keep the network reliability at 100%. The rules together mean that a successful transmission can occur only when a node has a packet to transmit and its target node has an empty buffer. This is a completely distributed method to prevent packets from getting too closely spaced, and, in consequence, efficiently regulate the traffic flow in the network.

Rule 1 ensures that relay nodes may have at most one packet in their buffer and is favorable for the following reasons:

- Keeping buffer sizes small can prevent the mean and the variance of the *in-network* end-to-end delay[1] from getting excessive. Indeed, when buffer capacities are large, several packets may get stacked in them, especially when the link quality is poor, thus transportation of packets across the links get delayed. In other words, the packet delays are much more tightly controlled when the buffer sizes are smaller. Thus, depending on the time a packet spends in its buffer, the source node can judiciously decide whether to drop it or not.

  To illustrate this effect, we plot the empirical mean (solid lines) and variance (dashed lines) of the (in-network) end-to-end delay versus the link reliability p_s for a CSMA/CA-based line network with 10 relays (see Fig. 1). For the simulation, we assumed the source node to be always backlogged, and the backoff time to be exponential with a mean of one time slot. Notice the increase in the mean and the variance of the end-to-end delay with increasing buffer size. For general MAC schemes where nodes are likely to interfere with each others' transmissions, the link reliabilities are even smaller, thus larger buffers affects network delays more drastically.
- Keeping buffer sizes small reduces hardware cost and energy consumption.

B. MAC Schemes

For our analysis, we consider two slotted channel access methods that are tractable: a modified version of the traditional TDMA which we call *randomized TDMA (r-TDMA)* and *slotted ALOHA*. In r-TDMA, the transmitting node in each time slot is chosen uniformly randomly from the set of all nodes (with probability (w.p.) $1/(N+1)$) instead of being picked in an ordered fashion. In the ALOHA-based network, in each time slot, each node having a packet independently transmits with a certain probability of contention q.

III. Related Work

A. Literature Review

The delay and throughput performances of the classical TDMA, spatial TDMA, ALOHA and several other MAC schemes have been extensively studied for point-to-point links, often using queueing-theoretic approaches (e.g. [3], [4]). Queueing theory has also been used to characterize the throughput and delay performance of flows involving multiple hops (e.g. [5], [6]). However, such analyses are less tractable and often yield only approximate results. In order to circumvent these issues, authors have considered very small [7] or infinitely large [8] networks. Moreover, previous studies have either considered unlimited buffer capacities [9] or neglected queueing delays in the system [10], both of which are not realistic assumptions. In this paper, we use existing results from the TASEP literature to derive exact analytical results on the throughput and delay performance of wireless line networks with an arbitrary number of nodes. The TASEP-based framework also has the advantage of obviating the often unwieldy queueing theory-based analysis.

Since we consider relays with unit buffers, the queueing delays at the relay nodes are zero, and we only need to consider access and retransmission delays. The benefits of keeping relay buffer sizes equal to unity has been studied earlier in literature. [11] considers a buffering policy similar to the one described earlier in this paper, and proposes several amendments to the MAC layer, such as the notion of shadow packets to stabilize the system and achieve the optimal throughput. In [12], the authors show that buffering and network coding implemented at the source node can lead to comparable packet drop rates as to buffering at every intermediate router. In the case of large networks with multiple links, the coding-based scheme can also provide buffer gains. In [13], it is proven that for a line network, the optimal scheduling algorithm that minimizes the end-to-end buffer usage gives preference to serving links closer to the destination. Hence, much of the buffering should occur at the source node.

B. An Overview of TASEPs with Open Boundaries

The TASEP refers to a family of simple stochastic processes used to describe the dynamics of self-driven systems

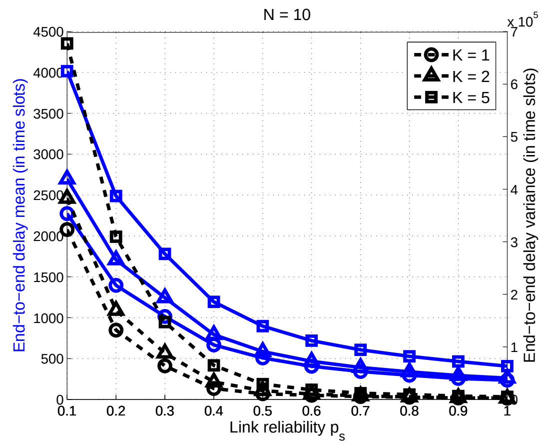

Fig. 1. The empirical mean (solid lines) and variance (dashed lines) of the in-network end-to-end delay in CSMA-based wireless network versus the link reliability p_s, for different values of K, the buffer size at nodes. The larger the buffer capacities of the nodes, the higher are the delay mean and variance.

[1] In this paper, we are only interested in the in-network delay, defined as the delay incurred by the packet from the time it arrives at the head of the source node's queue to the time it is delivered. In other words, we do not consider the queueing delay at the source node.

with several interacting particles and is a paradigm for non-equilibrium systems [2]. The classical 1D TASEP model with open boundaries is defined as follows. Consider a system with $N+1$ sites, numbered 0 to N. Site 0 is taken to be the source that injects particles into the system. The model is said to have open boundaries, meaning that particles are injected into the system at the left boundary (site 1) and exit the system on the right boundary (site N). The *configuration* of site i, $1 \leq i \leq N$ at time t is denoted by $\tau_i[t]$, which can only take values in $\{0, 1\}$, i.e., each site $1 \leq i \leq N$ may either be *occupied* (denoted as $\tau_i[t] = 1$) or *empty* (denoted as $\tau_i[t] = 0$). The source, however, is taken to be always occupied ($\tau_0[t] \equiv 1, \forall t > 0$).

In the discrete-time version of the TASEP, the movement of particles is defined to occur in time steps. Specifically, let $(\tau_1[t], \tau_2[t], \ldots, \tau_N[t]) \in \{0, 1\}^N$ denote the configuration of the system in time slot t. In the subsequent time slot $t + 1$, a set of sites is chosen at first, depending on the *updating procedure*. Then, for every site picked, if it contains a particle and the neighboring site on its right has none, then the particle hops from that site to its neighbor with a certain probability p. This way, the particles are transported from site 0 through the system until their eventual exit at site N. The movement of particles to the right is equivalent to the movement of *holes* (or empty sites) to the left. This *particle-hole symmetry* leads to some interesting system dynamics, as we shall see later.

In this paper, we focus on the following two commonly considered TASEP updating procedures:

1) *Random-sequential TASEP*: In each time step, a single site is uniformly randomly picked (w.p. $1/(N + 1)$) for transmission, and particle hopping is performed as per the aforementioned rules.
2) *Parallel TASEP*: The updating rules are simultaneously applied to all the sites, i.e., in each time slot, all particles having an empty site to their right jump concurrently.

For both these updating procedures, it is known that in the long time limit ($t \to \infty$), the TASEP system attains a *steady state* wherein the rate of particle flow becomes a constant [2].

It is apparent from the description of the TASEP model that it exhibits a similarity to wireless line networks. The sites can be taken to represent the relay nodes and the particles the packets. The hopping probability p is analogous to the link reliability p_s while the exclusion principle models the unit buffer size at the relay nodes. Also, the updating procedure in the TASEP model relates to the MAC scheme in the wireless line network. The condition $\tau_0[t] = 1, \forall t$, is integrated into the system by assuming that the source node is backlogged, i.e., it always has packets to transmit. Fig. 2 depicts the TASEP-equivalence of the line network flow, wherein we assume that the backlogged source has a large buffer and regulates the packet flow into a TASEP model.

IV. Throughput and Delay Analysis for the r-TDMA-based Line Network

In this section, we characterize the throughput and average in-network delay behavior of r-TDMA-based line network at steady state (as $t \to \infty$). Noting that the r-TDMA scheme is analogous to the random sequential update, we use existing results from the random sequential TASEP literature for our analysis. We also apply our findings to studying the interesting short-hop versus long-hop routing problem.

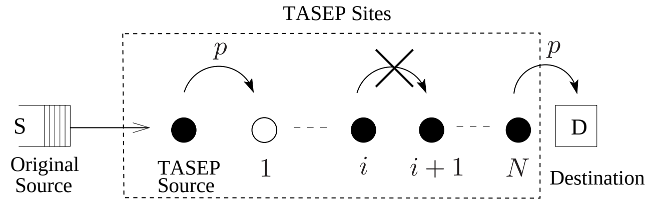

Fig. 2. The wireless line network is modeled as a source node with a large buffer connected to the TASEP particle flow model with $N + 1$ sites, each with a buffer size of unity. The hopping probability across each link is equal to p. Filled circles indicate occupied sites and the rest indicate holes. Jumping from site j to k is possible only if the configuration $\{\tau_j, \tau_k\}$ is {1,0}. In the above example, hopping is not possible between sites i and $i + 1$.

Since there is no interference in the r-TDMA-based network and the fading power is exponentially distributed, we obtain

$$p_s = \Pr[\text{SNR} > \Theta] = \exp\left(-\Theta N_0 l^{\gamma}\right). \tag{1}$$

Also, since the links are spaced equally, the success probability across any link is the same. This is equivalent to taking $p = p_s$ in the corresponding TASEP model.

A. Steady State Occupancies

We begin our analysis by studying the steady state *occupancy* of a node i, defined as the probability that it is occupied at steady state, i.e., $\mathbb{P}(\lim_{t\to\infty} \tau_i[t] = 1)$. Hereafter, we use the simplified notation $\tau_i :\triangleq \lim_{t\to\infty} \tau_i[t]$. Now, since τ_i can take values only in $\{0, 1\}$, $\mathbb{P}(\tau_i = 1) = \mathbb{E}\tau_i$ and $\mathbb{P}(\tau_i = 0) = 1 - \mathbb{E}\tau_i$. In other words, the occupancy of node i is the same as the average number of packets at the i^{th} node's queue. From [14, Eqn. 48], we have for $0 \leq i \leq N$,

$$\mathbb{E}\tau_i = \frac{1}{2} + \frac{1}{4} \frac{(2i)!}{(i!)^2} \frac{(N!)^2}{(2N + 1)!} \frac{(2N - 2i + 2)!}{[(N - i + 1)!]^2} (N - 2i + 1). \tag{2}$$

Surprisingly, the node occupancies are independent of p_s. Also, notice the particle-hole symmetry, i.e., $\mathbb{E}\tau_i = 1 - \mathbb{E}\tau_{N+1-i}$. Thus, $\sum_{i=0}^{N} \mathbb{E}\tau_i = 1 + N/2$. In a system with an odd number of relays, the middle relay has an occupancy of exactly $1/2$. Fig. 3 shows the occupancies $\mathbb{E}\tau_i$ for a multihop network with $N = 10$ relay nodes.

B. Steady State Throughput

We now derive the throughput of the line network at steady state, defined as the average number of packets successfully delivered (to the destination) in a unit step of time.

Corollary 4.1: For the r-TDMA-based line network with N nodes, the throughput at steady state is

$$T = \frac{p_s(N + 2)}{2(N + 1)(2N + 1)}. \tag{3}$$

Proof: At any instant of time, node N's buffer contains a packet w.p. τ_N; furthermore, it is picked for transmission w.p.

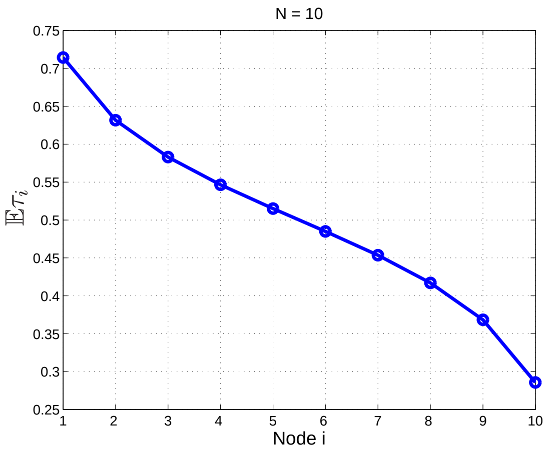

Fig. 3. The steady state occupancy of each relay node for a r-TDMA-based multihop network with $N = 10$ relays. Notice the particle-hole symmetry, i.e., $\mathbb{E}\tau_i = 1 - \mathbb{E}\tau_{N+1-i}$.

$1/(N+1)$, and the transmission is successful w.p. p_s. Thus, the throughput of the line network is simply

$$T = p_s \mathbb{E}\tau_N/(N+1). \tag{4}$$

Using (2) in (4), we obtain the desired result. ■

The system throughput at steady state is proportional to the link reliability and upper bounded by $p_s/4$, but decreases with increase in the system size: $T \sim p_s/(4N)$ for[2] $N \gg 1$. Also, since the reliability of the network is 100%, the rate of packets across each link is the same, and equal to (3).

C. Average End-to-End Delay at Steady State

Corollary 4.2: For the wireless multihop network with N relays running the r-TDMA scheme, the average delay experienced by a packet at node i is

$$\mathbb{E}D_i = \frac{2(N+1)(2N+1)\mathbb{E}\tau_i}{(N+2)p_s}, \quad 0 \le i \le N, \tag{5}$$

and consequently, the average in-network end-to-end delay is

$$\mathbb{E}D_{\text{e2e}} = \sum_{i=0}^{N} \mathbb{E}D_i = \frac{2N^2+3N+1}{p_s}. \tag{6}$$

Proof: Recall that the rate of packet flow across each node is equal to T, and that the average number of packets at node i, $0 \le i \le N$ is $\mathbb{E}\tau_i$. From Little's theorem [15], the average delay at node i is simply $\mathbb{E}\tau_i/T$. ■

We see that the average end-to-end delay is proportional to the node occupancies and inversely proportional to the link reliability. Also, it is interesting to note that the product of throughput and delay is $1 + N/2$, which is independent of p_s.

For large N, we immediately see from (6) that

$$\mathbb{E}D_{\text{e2e}} \sim 2N^2/p_s. \tag{7}$$

The average end-to-end delay grows quadratically with the number of relay nodes N.

D. The Short-hop versus Long-hop Routing Problem

We now present a simple application of our results: the short-hop versus long-hop routing problem [16] in long ($N \gg 1$) regular r-TDMA-based wireless networks. Specifically, we determine if it is beneficial to route over many short hops or a smaller number of longer hops. The metrics we use for comparison are the average end-to-end delay and throughput.

To this end, let us suppose that communication occurs only across nodes that are in general, m hops ($1 \le m \le N$) apart. Manipulating (1), it is straightforward to see that

$$p_s = \exp\left(-\Theta N_0 (ml)^\gamma\right).$$

We now determine the optimum spacing between the communicating hops, m_{opt}, that minimizes the average end-to-end delay for this general line network. Since there are N/m relays now, we have from (7) (assuming that N is a multiple of m),

$$\mathbb{E}D_{e2e} \sim 2(N/m)^2/p_s = 2N^2 \exp\left(\Theta N_0 (ml)^\gamma\right)/m^2, \tag{8}$$

Upon differentiating (8), we obtain[3]

$$m_{\text{opt}} = \frac{1}{l}\left(\frac{2}{\Theta N_0 \gamma}\right)^{1/\gamma}, \tag{9}$$

which is independent of N.

The values of m_{opt} (9) for several values of γ and Θ is plotted in Fig. 4. Depending on the value of the SNR threshold, routing needs to be performed over longer or shorter hops in order to keep the packet delay minimal.

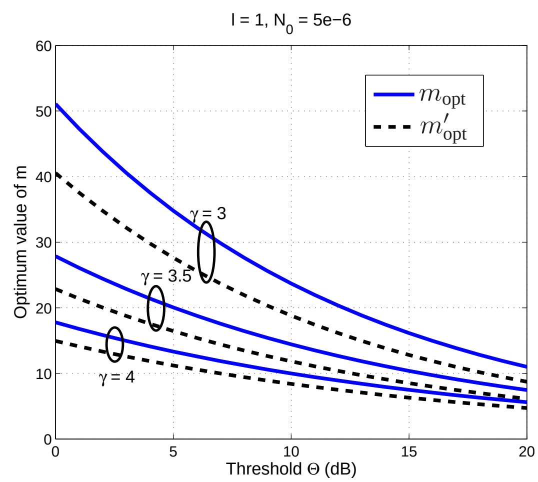

Fig. 4. Delay-optimum hop spacing m_{opt} (9) and throughput-optimum hop spacing m'_{opt} (11) versus Θ for different values of the path loss exponent.

Likewise, let m'_{opt} denote the optimum value of the spacing between hops for which the network throughput is maximized. We can express the throughput for the N/m-relay system as

$$T \sim p_s/\left(4N/m\right) = m\exp\left(-\Theta N_0 (ml)^\gamma\right)/4N, \tag{10}$$

[2]The notation $f(n) \sim g(n)$ means that the ratio $f(n)/g(n)$ approaches 1 asymptotically (as $n \to \infty$).

[3]We allow m to assume any real value here. In practice, m_{opt} will be rounded up or down to an integer.

which is maximized at

$$m'_{\text{opt}} = \frac{1}{l}\left(\frac{1}{\Theta N_0 \gamma}\right)^{1/\gamma}. \tag{11}$$

We see that $m_{\text{opt}}/m'_{\text{opt}} = 2^{1/\gamma}$. The values of m'_{opt} are also depicted in Fig. 4.

V. Throughput and Delay Analysis for the Slotted ALOHA-based Line Network

In this section, we employ existing results from the TASEP particle model with parallel update to analytically derive the buffer occupancies, throughput and average end-to-end delay for the slotted ALOHA-based network at steady state. Additionally, we apply our findings to determine the optimum contention parameter that minimizes the average end-to-end delay in long wireless line networks.

A. Steady State Buffer Occupancies

Suppose that the link reliabilities[4] are each equal to p_s. Also, let q denote the contention probability, i.e., in each time slot, nodes having a packet independently transmit w.p. q or stay idle w.p. $1-q$. We can take the effective hopping probability in the corresponding parallel TASEP model to be $p = qp_s$. Then, the steady state occupancies are given by [17, Eqn. 10.16]

$$\mathbb{E}\tau_i = \frac{(1-qp_s)\sum_{n=0}^{N-i} B(N-n)B(n) + qp_s B(N)}{B(N+1) + qp_s B(N)}, \tag{12}$$

where $B(0) = 1$, and

$$B(k) = \sum_{j=0}^{k-1} \frac{1}{k}\binom{k}{j}\binom{k}{j+1}(1-qp_s)^j, \quad k > 0.$$

The steady state occupancies depend nontrivially on p (and hence, on q and p_s) as depicted in Fig. 5. Also, owing to the particle-hole symmetry, we have $\mathbb{E}\tau_{\lceil (N+1)/2\rceil} = \mathbb{E}\tau_{\lfloor (N+1)/2\rfloor}$, and $\sum_{i=0}^{N} \mathbb{E}\tau_i = 1 + N/2$. For the special case $p = 1$, i.e., $q = p_s = 1$, the steady state configuration of each node alternates between ones and zeros, and the occupancy of each relay node is exactly $1/2$.
When $N \gg 1$, $\mathbb{E}\tau_1 = \left(2p - 1 + \sqrt{1-p}\right)/2p$ and $\mathbb{E}\tau_N = \left(1 - \sqrt{1-p}\right)/2p$ [17]. Also, the occupancy in the bulk is approximately equal to $1/2$, i.e., $\mathbb{E}\tau_i \approx 1/2$ for $1 < i < N$.

B. Steady State Throughput

The steady state throughput is simply given by $T = qp_s\mathbb{E}\tau_N$, because the probability that a particle is successfully delivered to the destination in any given time step is $qp_s\tau_N$. Note that since the rate of packet flow across all links are the same, the throughput could also have been derived using the equivalent expression $T = qp_s\mathbb{E}[\tau_i(1-\tau_{i+1})], 0 \le i \le N$. From (12), the throughput of the slotted ALOHA-based line network is

$$T = qp_s\mathbb{E}\tau_N = \frac{qp_s B(N)}{B(N+1) + qp_s B(N)}. \tag{13}$$

[4]In general, the link reliability p_s is a function of the contention probability q, since the interference in the network depends on q. This will be discussed in Subsection V-D.

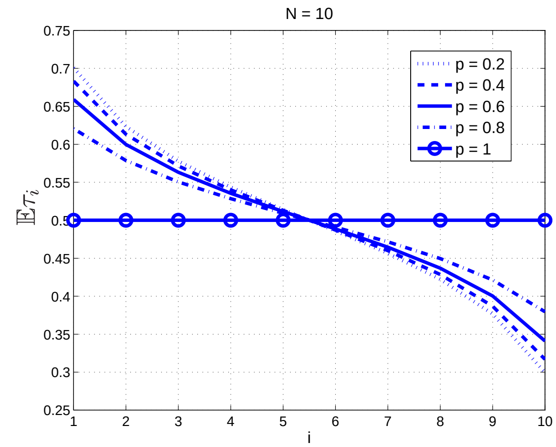

Fig. 5. The occupancies for the parallel TASEP particle flow model for several values of p. Unlike the r-TDMA case (see Fig. 3), $\mathbb{E}\tau_i$ depends on p.

For $N \gg 1$, the network throughput at steady state is

$$T \sim \left(1 - \sqrt{1 - qp_s}\right)/2. \tag{14}$$

C. Average Steady State Delay

Corollary 5.1: For the ALOHA-based wireless line network with link reliability p_s, the average steady state delay D_i of a packet at node i is equal to

$$\mathbb{E}D_i = \mathbb{E}\tau_i/(qp_s\mathbb{E}\tau_N), \quad 0 \le i \le N. \tag{15}$$

Consequently, the average end-to-end delay is

$$\mathbb{E}D_{\text{e2e}} = \frac{1}{qp_s\mathbb{E}\tau_N}\sum_{i=1}^{N}\mathbb{E}\tau_i = \frac{N+2}{2qp_s\mathbb{E}\tau_N}. \tag{16}$$

Proof: The proof is identical to the one used to derive the delay across the r-TDMA-based network (see Corollary 4.2), and follows directly from Little's theorem [15]. ∎
For large N, we have

$$\mathbb{E}D_{\text{e2e}} \sim \frac{N+2}{1 - \sqrt{1 - qp_s}}. \tag{17}$$

As in the r-TDMA-based network, $T \times \mathbb{E}D_{\text{e2e}} = 1 + N/2$ here. For the special case $q = p_s = 1$, every alternate node transmits successfully in each time slot; the throughput is equal to $1/2$, and the delay at each hop is 2.

D. Optimizing the Contention Probability in Long ALOHA-based Line Networks

Consider a long ($N \gg 1$) ALOHA-based wireless line network employing the modified transmission scheme. For small q, nodes hold on to packets for a long time before transmitting them, which results in a long delay. Likewise, for high q, the interference in the network is high and the delay is large. In this subsection, we study the interesting question of how to choose the optimum q that minimizes the end-to-end delay at steady state. An alternative problem is choose the value of q that maximizes the steady state throughput.

We assume that the system is interference-limited, thus $p_s = \Pr[\text{SIR} > \Theta]$. Now, from [18], the success probability p_s for the considered line network model is well-approximated by

$$p_s \approx \exp(-qc/2), \tag{18}$$

where $c = \pi\Theta^{1/\gamma}/\sqrt{\gamma/2} - 1$.
Using (18) in (17), we obtain

$$\mathbb{E}D_{\text{e2e}} \propto \left(1 - \sqrt{1 - q\exp(-qc/2)}\right)^{-1}. \tag{19}$$

Differentiating (19) and noting that $0 \leq q \leq 1$, the optimum value of the contention parameter that minimizes the average end-to-end delay is obtained as

$$q_{\text{opt}} = \min\{1, 2/c\}. \tag{20}$$

From (14), we see that $T \propto 1 - \sqrt{1 - q\exp(-qc/2)}$, thus q_{opt} maximizes the steady state throughput as well. Fig. 6 plots analytical values of q_{opt} (20) versus the SIR threshold Θ, for several values of γ (dashed lines). It also shows empirical values of q_{opt} obtained via simulation (solid lines), which are seen to match the analytical values closely.

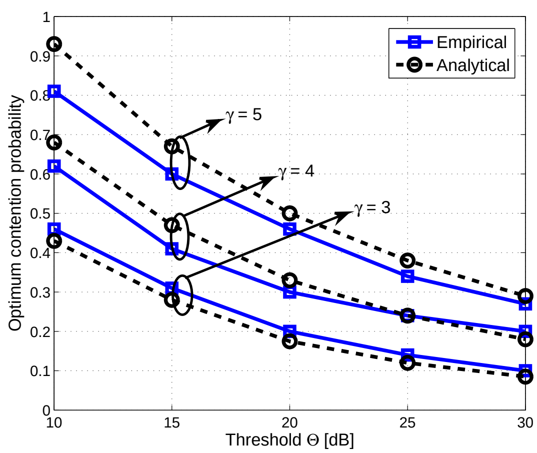

Fig. 6. The analytical (dashed lines) and empirical (solid lines) values of the optimum contention parameter q_{opt} that minimizes the end-to-end delay (as well as maximizes the steady state throughput) versus Θ for different values of γ, in a long ($N \gg 1$) regular ALOHA-based wireless network.

VI. Summary and Concluding Remarks

We propose a modified transmission policy for wireless networks that helps regulate the flow of packets in a completely decentralized manner. Using known results from statistical mechanics, in particular, the TASEP particle flow model, we characterize the steady state end-to-end delay and throughput performances of multihop line networks running the r-TDMA and slotted ALOHA MAC schemes. We also extend the results derived to long networks and provide applications to important wireless networking problems.

The TASEP particle-flow model permits the application of statistical mechanics to wireless networking. It helps provide closed-form expressions for the average end-to-end delay and throughput of the multihop line network and has the advantage of obviating the cumbersome queueing theory-based analysis. Furthermore, the results obtained are scalable with the number of nodes and thus can provide helpful insights into the design of wireless networks. We wish to promote TASEPs as a useful tool to analyze the performance of ad hoc networks and hope that this introductory work instigates interest in solving other relevant wireless networking problems employing ideas from statistical mechanics.

Acknowledgments

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