

Throughput-Delay-Reliability Tradeoffs in Multihop Networks with Random Access

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Abstract—In ad hoc networks, performance objectives are often in contention with each other. Indeed, due to the transmission errors incurred over wireless channels, it is difficult to achieve a high rate of transmission in conjunction with reliable delivery of data and low latency. In order to obtain favorable throughput and delay performances, the system may choose to compromise on its reliability and have nodes forcibly dropping a small fraction of packets. The focus of this paper is on the characterization of tradeoffs between the achievable throughput, end-to-end delay and reliability in wireless networks with random access.

We consider a multihop ad hoc network comprising several source-destination pairs communicating wirelessly via the slotted ALOHA channel access scheme. Employing ideas from statistical mechanics, we present an analytical framework for evaluating the throughput, end-to-end delay and reliability performances of the system. The main findings of this paper are (a) when the system is noise-limited, dropping a small fraction of packets in the network leads to a smaller end-to-end delay though the throughput suffers as well, and (b) when the system is interference-limited, however, there exist regimes where dropping a few packets in the network may actually reduce the end-to-end delay as well as increase the system throughput. We also present some empirical results which corroborate the results obtained analytically.

I. INTRODUCTION

A. Motivation

An ad hoc network comprises several source-destination node pairs that can communicate wirelessly in a decentralized fashion owing to their self-organizing capabilities. The nodes are energy-limited, thus they typically employ multihop routing, wherein relay nodes assist the delivery of packets across nodes lying far away from each other. Ad hoc networks are touted as being extremely promising for several reasons, such as being easily and rapidly deployable and reconfigurable, and also for the fact that they lack single points of failure compared to traditional network architectures, such as cellular networks and WLANs. In spite of not having a centralized infrastructure, these systems are intended to provide reliable broadband services across multiple hops, for example in mesh networks [1].

Performance goals in wireless networks, however, often conflict with one another. For instance, it is hardly possible to guarantee a high rate of transmission, i.e., throughput (or a small end-to-end delay) in conjunction with highly reliable packet delivery, due to the random transmission errors caused by the unpredictable behavior of the wireless channel. In

particular, when the link qualities in the system are poor, packets require to be retransmitted several times across hops in order to assure reliable end-to-end delivery. This, however, leads to queueing of packets at the relay nodes, resulting in an unreasonably large average end-to-end delay, as well as a low rate of transmission. Evidently, there exist tradeoffs between the throughput, the end-to-end delay and reliability of ad hoc networks.

In scenarios where reliable delivery of packets is not very critical, a viable solution to balance end-to-end delay and reliability is to have the nodes forcibly drop a small fraction of packets. That way, even though the network reliability is reduced slightly, the queueing delay of packets can be lessened considerably. In order to determine the optimal operating point of the system and effectively study the achievable quality of service offered by the network, it is important to analyze the throughput-delay-reliability (TDR) tradeoffs, which is the primary focus of this paper.

We consider a multihop wireless network consisting of several source-destination pairs communicating with each other employing the slotted ALOHA channel access mechanism, and present an analytical framework that helps quantify the TDR performances of the system. We find that while in the noise-limited regime, dropping a small fraction of packets in the network leads to a smaller end-to-end delay at the cost of reduced throughput, in the interference-limited scenario, dropping a few packets in the network can help mitigate the interference in the network leading to an increased throughput. We also present some empirical (simulation-based) results which closely match the values obtained analytically.

B. Related Work

Scaling laws governing the tradeoff between throughput and delay in wireless networks comprising several users are a fairly well-investigated topic [2], [3]. More recently, the effect of dropping packets on the delay and throughput performance of single-hop wireless networks has been studied [4]. However, little work exists towards characterizing TDR tradeoffs in the context of multihop wireless network flows comprising a finite number of relays.

In [5], the authors evaluate the delay-reliability tradeoff in a wireless line network for a bounded delay packet dropping strategy employing queueing theory. However, their analysis

neglects the dependence of the link success probabilities on the packet dropping event. In [6], the author uses the notion of transmission capacity to characterize the TDR tradeoffs in wireless networks employing a packet dropping scheme based on limited retransmissions. However, it is assumed that all nodes in the network are backlogged, i.e., always have packets to transmit.

In this work, we use some ideas from the literature on statistical mechanics, in particular the totally asymmetric simple exclusion process, a particle flow model, and the mean-field approximation. Employing these tools, we present a simple framework to analyze ad hoc networks, which also has the advantage of obviating the often-unwieldy queuing theory-based analysis (that is typically used to study multihop networks). To the best of our knowledge, this is the first attempt at quantifying the TDR performances of ALOHA-based multihop wireless networks, all together, whilst explicitly taking into consideration the nodes' buffer occupancies and the effect of dropping packets on the interference in the network.

The rest of the paper is organized as follows. Section II describes the considered ad hoc network model, and also outlines the channel access, routing and buffering schemes considered in this paper. Section III studies the TDR tradeoffs in ad hoc networks, treating the noise-limited and interference-limited regimes separately. Section IV concludes the paper.

II. SYSTEM MODEL

We consider an ad hoc network comprised of an infinite number of source nodes, each of which intends to establish a (in general, multihop) flow of packets to a certain destination node lasting over an infinite duration of time. This framework is suitable for modeling ad hoc networks since the aggregate traffic in an ad hoc network can always be decomposed into several unicast multihop flows. The distribution of source nodes is assumed to be a homogeneous Poisson point process (PPP) on the infinite plane \mathbb{R}^2 with density δ . Additionally, the network consists of a countably infinite population of other nodes (potential relays and destinations) arranged as a homogeneous PPP with density $1-\delta$. Thus, the total density of the network is (without loss of generality) equal to unity. For each source node, the destination node is chosen at a random orientation, and at a random finite distance.

A. Routing Strategy

It is assumed that each source knows its own location and the direction towards its intended destination. Packets are then routed in a general manner as follows: each node that receives a packet relays it to its n^{th} -nearest-neighbor ($n \geq 1$) in a sector of angle $\phi \in [0, \pi]$, i.e., the next-hop node is the n^{th} -nearest-neighbor that lies within $\pm\phi/2$ of the axis to the destination. Fig. 1 (top) illustrates the case of nearest-neighbor ($n = 1$) routing.

A sample realization of the system model comprising several source-destination pairs is shown in Fig. 1 (bottom) with $\delta = 0.05$ and $\phi = \pi/2$. In the figure, each destination is assumed to be located 5 nearest-neighbor ($n = 1$) hops

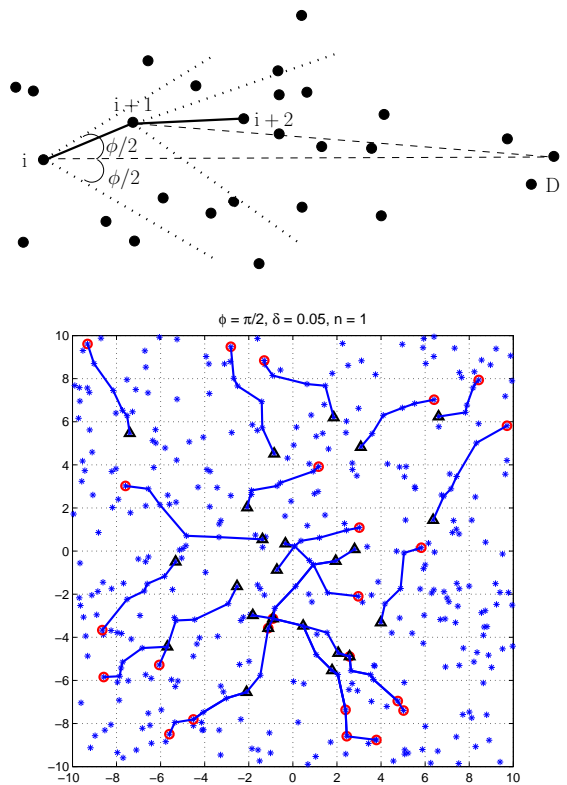


Fig. 1. (Top) Illustration of nearest-neighbor routing in a sector of angle $\pm\phi/2$ along the axis to the destination for an arbitrary flow. The packet is routed from node i to node $i+1$, which then relays it to node $i+2$, and so on. (Bottom) A sample realization of the ad hoc network model comprising several flows (marked by thick solid lines). The circles depict the sources, while the triangles represent destinations. In this illustration, each destination is assumed to be located 5 nearest-neighbor ($n = 1$) hops away from its corresponding source node. The density of source nodes is $\delta = 0.05$, and the routing sector angle is $\phi = \pi/2$.

away from its corresponding source. Note that in general, the same common relay node may be a part of multiple flows, in particular when δ is large.

We take that all nodes use the same frequency band such that simultaneous transmissions interfere with each other. Furthermore, no power control is employed; we simply assume that the transmit power at each transmitting node is equal to unity. Also, we model the attenuation in each link as the product of a large-scale path loss with exponent γ and an i.i.d. Rayleigh fading component. Time is slotted, and transmission attempts occur at slot boundaries. Now, let $\Phi = \{x_i\}$ denote the set of transmitters in an arbitrary time slot. Then, the total received power at location y on the plane in that time slot is

$$I_{\Phi}(y) = \sum_{x \in \Phi} G_{xy} g(x-y),$$

where G_{xy} denotes the (power) fading gain of the wireless link between x and y , and $g(z) = \|z\|^{-\gamma}$. We define the transmission of a packet from a node located at x_j to another located at y to experience an outage if and only if the instantaneous signal-to-interference and noise-ratio (SINR) at

y is smaller than a threshold Θ , i.e., the probability of a successful transmission at the receiver at y is given by

$$p_s = \mathbb{P}\left(\frac{G_{x_j y} \|x_j - y\|^{-\gamma}}{N_0 + I_{\Phi \setminus \{x_j\}}(y)} \geq \Theta\right), \quad (1)$$

where N_0 denotes the noise (AWGN) variance.

B. Buffering and Transmission Policy

We now introduce a buffering and transmission policy for each flow in the network, which obeys the following two rules.

- 1) All the buffering in the network is performed at the source nodes, while each relay node has a buffer size of unity (for each flow it is associated with). Thus, all the queueing occurs at the source, while relay nodes may hold at most one packet (per flow). We also take that the source nodes are backlogged, i.e., they always have packets to transmit.
- 2) Incoming transmissions are not accepted by relays if their buffer already contains a packet.

The two aforementioned rules together mean that a successful transmission may occur only when a node has a packet and its target node's buffer is empty. Employing this transmission scheme prevent packets from getting closely spaced, and in consequence, efficiently regulates traffic along the flows in a completely distributed manner. Furthermore, it prevents the end-to-end delay from getting excessive since packets never get stacked up at buffers, in particular when the link reliabilities are small. A more detailed discussion of the benefits of using this single-buffer transmission scheme can be found in our previous work [7] (and the references therein).

We remark that since the distribution of nodes is homogeneous, it is sufficient to analyze a “typical” flow in the system. Fig. 2 depicts the schematic of a representative flow in the network across N relays. The source node is numbered 0, while the relay nodes are numbered 1 through N . All the results in this paper are obtained for an “average” network, that is the one obtained upon averaging over all possible realizations of the channels and the underlying point processes.

For a typical flow across N relay nodes, we denote the *occupancy* of node i 's buffer (corresponding to that flow) in time slot t by $\tau_i[t]$, $0 \leq i \leq N$. We take $\tau_i[t] = 1$ when node i 's buffer is occupied, i.e., it has a packet, and $\tau_i[t] = 0$ otherwise. Since the source node is always backlogged, $\tau_0[t] = 1, \forall t$. Note that a packet may successfully hop between nodes i and $i+1$ in time slot t only if $\{\tau_i[t], \tau_{i+1}[t]\} = \{1, 0\}$, and furthermore, if its transmission is successful, which happens with probability (w.p.) p_s .

C. MAC Scheme: slotted ALOHA

We assume that transmissions in the network are completely uncoordinated; the transmission scheme is slotted ALOHA. Accordingly, in each time slot, every node *having a packet* independently transmits with some (contention) probability q or remains idle w.p. $1 - q$.

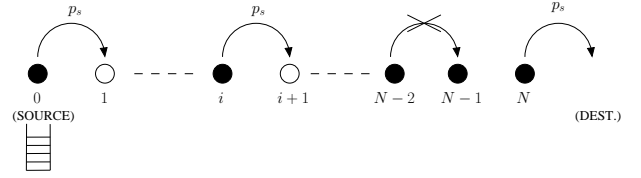


Fig. 2. Depiction of a “typical” flow with the link reliabilities p_s . The source node (numbered 0) is always backlogged and has a large buffer that feeds packets required to be delivered. Relays $1, \dots, N$ have buffer sizes of unity. In the above figure, filled circles represent nodes with packets while empty circles indicate nodes with empty buffers. For this snapshot, we see that a transmission from node $N-2$ to node $N-1$ will fail because $\tau_{N-1} = 1$.

D. Performance Metrics

We are interested in the performance of the ad hoc network in its *steady state* (as $t \rightarrow \infty$). The performance of the system is characterized based on three end-to-end metrics, throughput, mean end-to-end delay and reliability, each evaluated for a typical flow at steady state. They are formally defined as follows.

- The per-flow **throughput** T , is defined as the average number of packets successfully delivered (to the destination) in unit time, along a typical flow in the network.
- The **mean end-to-end delay**, D , is defined as the average number of time slots it takes for the packet at the head of the source node¹ to successfully hop to the destination.
- The **end-to-end reliability** R is defined as the fraction of packets generated at the source that are eventually successfully delivered. By definition, $0 \leq R \leq 1$.

III. TDR CHARACTERIZATION FOR THE ALOHA-BASED WIRELESS NETWORK

In this section, we introduce a framework based on mean-field theory that we will employ to characterize the TDR tradeoffs for the considered ad hoc network model. For analytical tractability, we neglect the interactions between flows that occur via common relays². We treat the noise-limited and interference-limited regimes separately.

A. The Noise-limited Regime

We first consider the scenario where the noise power in the network is much stronger than the interference. This occurs, for instance, when the source density δ is small, or when the path loss exponent γ is large. Transmission success events across links are independent of the occupancies of other nodes in the network and occur w.p. $p_s = \mathbb{P}(\text{SNR} \geq \Theta)$.

1) **Case 1: $R = 1$:** We first consider the case with perfect reliability: all packets along each flow are retransmitted until they are successfully received. As described in our prior work [7], when $R = 1$, the transport of packets along each route

¹Note that we consider only the *in-network* delay since the source nodes are always backlogged.

²In other words, it is not possible for the same common relay node transmit or receive multiple packets (corresponding to different flows) simultaneously. This assumption is quite reasonable for small values of the contention parameter q or small δ (when the flows in the network themselves are sparse).

exhibits an analogy to a particle flow model in statistical mechanics, namely the totally asymmetric simple exclusion process (TASEP) [8]. We now use some known results from the TASEP literature to analyze the TDR characteristics.

As proven in [7], in the long-time limit ($t \gg 0$), the slotted ALOHA-based flow reaches a steady state wherein the probabilities $\mathbb{P}(\tau_i[t] = 0)$ (and $\mathbb{P}(\tau_i[t] = 1)$), $0 \leq i \leq N$, become temporally stationary (independent of time). Hereafter, we use the simplified notation $\tau_i \triangleq \lim_{t \rightarrow \infty} \tau_i[t]$ to denote the steady state occupancy of node i . Since $\tau_i \in \{0, 1\}$, we have $\mathbb{P}(\tau_i = 1) = \mathbb{E}\tau_i$ and $\mathbb{P}(\tau_i = 0) = 1 - \mathbb{E}\tau_i$. From [7, Eqn. 12], we have

$$\mathbb{E}\tau_i = \frac{(1 - qp_s) \sum_{n=0}^{N-i} B(N-n)B(n) + qp_s B(N)}{B(N+1) + qp_s B(N)}, \quad (2)$$

where $B(0) = 1$, and

$$B(k) = \sum_{j=0}^{k-1} \frac{1}{k} \binom{k}{j} \binom{k}{j+1} (1 - qp_s)^j, \quad k > 0.$$

The steady state occupancies depend non-trivially on the product term qp_s , as depicted in Fig. 3. Also, notice the *particle-hole symmetry*³, i.e., $\mathbb{E}\tau_i \equiv 1 - \mathbb{E}\tau_{N+1-i}$. Hence, in a system with an odd number of relays, the middle relay has an occupancy of exactly 1/2. The average number of packets in the flow at steady state is $\sum_{i=0}^N \mathbb{E}\tau_i = 1 + N/2$.

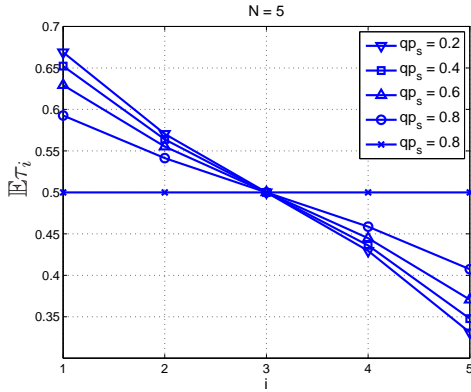


Fig. 3. Average node occupancies at steady state for an ALOHA-based flow with $N = 5$ and $R = 1$. Notice that they depend non-trivially on the product term qp_s . Notice the particle-hole symmetry: $\mathbb{E}\tau_i = 1 - \mathbb{E}\tau_{N+1-i}$.

The following lemma quantifies the throughput and mean end-to-end delay across a typical flow in closed-form.

Lemma 3.1: For an ALOHA-based line flow along N relays, the steady state throughput at full reliability ($R = 1$) is

$$T = \frac{qp_s B(N)}{B(N+1) + qp_s B(N)}, \quad (3)$$

while the average end-to-end delay is given by

$$D = (1 + N/2)/T. \quad (4)$$

³Particles (packets) moving towards the destination is equivalent to holes (empty buffers) moving towards the source.

Proof: Now, at any instant of time (in steady state), relay node N 's buffer has a packet w.p. τ_N ; furthermore, it transmits w.p. q , and the transmission succeeds w.p. p_s . Thus, the throughput is simply given by $T = qp_s \mathbb{E}\tau_N$, which is identical to (3).

Recall that at steady state, the average number of packets in the flow is $\sum_{i=0}^N \mathbb{E}\tau_i = 1 + N/2$. By Little's theorem [9], $D = \sum_{i=0}^N \mathbb{E}\tau_i / T$. ■

Evidently, $T \rightarrow 0$ while $D \rightarrow \infty$ as $p_s \rightarrow 0$. Also, as $N \rightarrow \infty$, $T \rightarrow (1 - \sqrt{1 - qp_s})/2$ [7, Eqn. 14]. It is interesting to note that irrespective of the values of q and p_s , the product of throughput and average delay for the $R = 1$ case is equal to the constant $1 + N/2$.

Fig. 4 plots a portion of the TDR region for the slotted ALOHA-based flow with $R = 1$ and $q = 0.2$, for different values of N ; they are essentially hyperbolas along the $R = 1$ axis. For each value of N , the curves are obtained by plotting the throughput (3) and delay (4) for different values of p_s .

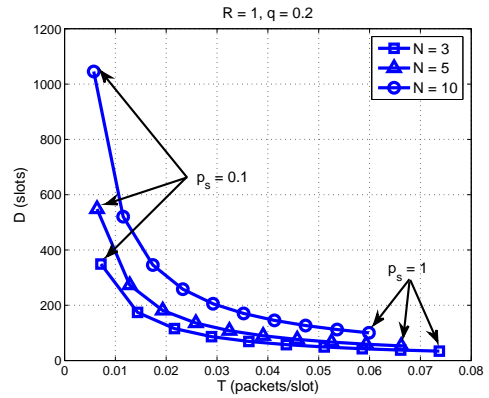


Fig. 4. A portion of the region (for $p_s = \{0.1, \dots, 1\}$) depicting the mean end-to-end delay versus the throughput for the ALOHA-based network, along the $R = 1$ axis. For each value of N , the TD curve is a hyperbola.

2) **Case 2: $R < 1$:** For the case with 100% reliability, the delay and throughput performances of the network are very poor, in particular when the link reliability p_s is small. In order to achieve favorable TDR tradeoffs, relay nodes may instead choose to drop a small fraction of packets. In the rest of this paper, we consider a *stochastic packet dropping scheme* which is straightforward to implement in a distributed fashion (with zero overhead). Accordingly, at every time slot, each node having a packet decides to drop the packet in its buffer or not stochastically (based on the toss outcome of a biased coin).

In this subsection, we evaluate the throughput, delay and reliability performances of the ALOHA-based network in the noise-limited regime. We show that dropping a small fraction of packets helps lessen the end-to-end delay (due to reduced queueing); however, it also results in a decreased flow throughput. We now provide a mean-field theory-based analytical framework for analyzing the TDR region of the wireless network.

Let ξ denote the packet dropping probability (or the bias

of the tossed coins). In an arbitrary time slot $t \rightarrow t + 1$, the following events can alter the configuration of node i .

- 1) If node i , $0 \leq i \leq N$ has a packet in its buffer,
 - it decides to drop its packet w.p. ξ .
 - it decides to transmit its packet w.p. $(1 - \xi)q$ (product of the packet-retention and the contention probabilities), and the packet hops to node $i + 1$ (if its buffer is empty) w.p. p_s .
- 2) If node $i - 1$ ($1 \leq i \leq N + 1$) has a packet in its buffer, it chooses to transmit (w.p. $(1 - \xi)q$), and its packet hops to node i (provided its buffer is empty) w.p. p_s .

In case 1), we have $\tau_i[t] = 1$ and $\tau_i[t + 1] = 0$. Likewise, the occurrence of 2) implies that $\tau_i[t] = 0$ while $\tau_i[t + 1] = 1$.

Following 1) and 2), the evolution of the node occupancies, τ_i for $1 \leq i \leq N$ takes the form

$$\begin{aligned} \Delta\tau_i[t] = & -\xi_i\tau_i - (1 - \xi_i)q_i\tau_i(1 - \tau_{i+1}[t])p_{s,i} \\ & + (1 - \xi_{i-1})q_{i-1}\tau_{i-1}[t](1 - \tau_i[t])p_{s,i-1}, \end{aligned} \quad (5)$$

where $\Delta\tau_i[t] = \tau_i[t + 1] - \tau_i[t]$, and $\{\xi_i, \xi_{i-1}\}$, $\{q_i, q_{i-1}\}$, and $\{p_{s,i}, p_{s,i-1}\}$ are all independent Bernoulli random variable pairs with means ξ , q and p_s respectively. At steady state, $\mathbb{P}(\lim_{t \rightarrow \infty} \tau_i[t] = 1) = 0$ becomes temporally stationary. In other words, $\mathbb{E} \lim_{t \rightarrow \infty} \Delta\tau_i[t] = 0$. From (5), this means that the set of equations,

$$-\xi\mathbb{E}\tau_i - (1 - \xi)qp_s[\mathbb{E}[\tau_i(1 - \tau_{i+1})] - \mathbb{E}[\tau_{i-1}(1 - \tau_i)]] = 0,$$

$1 \leq i \leq N$, has a solution. To solve for the mean node occupancies, we employ the *mean-field approximation*⁴, according to which the occupancies of the nodes are assumed to be uncorrelated⁵, i.e., $\forall i, j, \mathbb{E}[\tau_i\tau_j] = \mathbb{E}\tau_i\mathbb{E}\tau_j$. Then, for $1 \leq i \leq N$, (5) simplifies to

$$p_s(1 - \xi)q[\mathbb{E}\tau_{i-1}(1 - \mathbb{E}\tau_i) - \mathbb{E}\tau_i(1 - \mathbb{E}\tau_{i+1})] - \xi\mathbb{E}\tau_i = 0. \quad (6)$$

The steady state occupancies of nodes, $\mathbb{E}\tau_i$, $1 \leq i \leq N$ are evaluated by simultaneously solving this set of N non-linear equations, and may be performed numerically.

Fig. 5 plots the numerically evaluated mean occupancies of the nodes in the ALOHA-based flow, for some values of the packet dropping probability ξ . As expected, observe that the node occupancies decrease with increasing ξ . The empirical (simulation-based) values are also shown, and they closely match the values obtained numerically.

Asymptotics: When the number of nodes in the flow is large ($N \gg 1$), the set of non-linear equations (6) may be solved in closed form by explicitly considering the *quasi-continuum limit*. Accordingly, we fix the total length of the line network to a constant l , and take the lattice spacing constant to be $\epsilon = l/N$. Thus, for $N \gg 1$, $\epsilon \ll 1$, and the rescaled nodal position variable $x_i = il/N = i\epsilon$, $1 \leq i \leq N$ (hence, $1/N \leq x_i \leq 1$) is quasi-continuous. Without loss of generality, we may take the constant $l = 1$.

⁴The mean-field approximation is tight at small values of the 'effective' link reliability qp_s , and gets looser with increasing values of that product term [8].

⁵Since $\tau_i, \tau_j \in \{0, 1\}$, this also means that the occupancies are independent as $\mathbb{P}(\tau_i = 1, \tau_j = 1) = \mathbb{E}[\tau_i\tau_j] = \mathbb{E}\tau_i\mathbb{E}\tau_j = \mathbb{P}(\tau_i = 1)\mathbb{P}(\tau_j = 1)$.

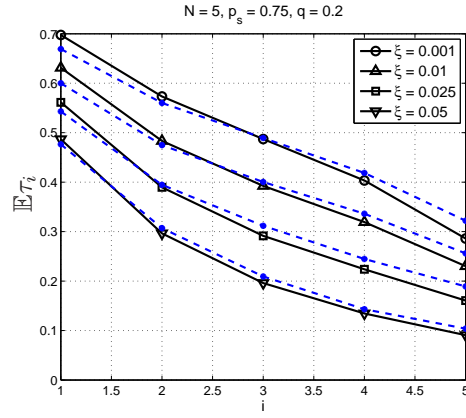


Fig. 5. Values of the mean node occupancies (solid lines) for several values of ξ with $N = 5$ relays, obtained upon numerically solving the set of equations (6). Values obtained empirically (dashed lines) are also plotted, and are seen to closely match the values obtained numerically.

From the Taylor series expansion for $\mathbb{E}\tau_{i-1}$ and $\mathbb{E}\tau_{i+1}$ in powers of ϵ , we obtain

$$\mathbb{E}\tau_{i\pm 1} = \mathbb{E}\tau_i \pm \epsilon \partial\mathbb{E}\tau_i / \partial x_i + O(\epsilon^2). \quad (7)$$

Employing (7) in (6) and neglecting terms with quadratic or higher orders in ϵ , we obtain

$$\partial\mathbb{E}\tau_i (2 - 1/\mathbb{E}\tau_i) \approx K \partial x_i, \quad 1 \leq i \leq N,$$

where $K = \xi / ((1 - \xi)qp_s\epsilon)$. Integrating both sides, we get

$$2\mathbb{E}\tau_i - \ln \mathbb{E}\tau_i \approx \frac{\xi i}{(1 - \xi)qp_s\epsilon} + C_i, \quad (8)$$

for some constants C_i , $1 \leq i \leq N$.

Note that setting $\xi = 0$ emulates the case wherein packets are never dropped ($R = 1$). Setting $\xi = 0$, we may write

$$C_i = 2\Delta_i - \ln \Delta_i, \quad 1 \leq i \leq N,$$

where we have from (2),

$$\Delta_i = \frac{(1 - qp_s) \sum_{n=0}^{N-i} B(N-n)B(n) + qp_s B(N)}{B(N+1) + qp_s B(N)}.$$

Now, the solution to (8) is expressible in terms of the Lambert W function [10] as

$$\mathbb{E}\tau_i \approx -\frac{1}{2} \mathcal{W} \left(-2 \exp \left(-\frac{\xi i}{(1 - \xi)qp_s} - C_i \right) \right), \quad (9)$$

where $\mathcal{W}(z)$ denotes the value of the Lambert W function at z . The Lambert W function, however, is a multi-valued function with two real branches, \mathcal{W}_0 and \mathcal{W}_{-1} . The branches merge at $z = -1/e$ where the Lambert W function takes the value -1 [10]. To evaluate (9), we need to choose the right branch of the Lambert W function.

To this end, we observe that the node occupancies monotonically decrease with proximity to the destination node. In other words, node 1 is the bottleneck node. This can be explained by noting that the destination is always willing to accept packets; thus the N^{th} relay node can empty its buffer at the highest

rate. However, the $N - 1^{\text{th}}$ relay needs the N^{th} relay to be empty to transmit its packet, so the likelihood that it will be occupied is higher when compared to node N , and so on.

Let $\psi_i = -2 \exp(- (K_i x + C_i))$. Evidently, ψ_i is always negative and $\psi_i \uparrow 0$ as $i \rightarrow \infty$. Now, for $z < 0$, $\mathcal{W}_0(z)$ is an increasing function of z , while $\mathcal{W}_1(z)$ decreases with increasing z [10]. Noting that $\mathbb{E}\tau_i$ is a decreasing function of i , it is possible to show after some manipulations that

$$\mathbb{E}\tau_i = \begin{cases} -1/2\mathcal{W}_{-1}(\psi_i) & \text{if } i \leq i^* \\ -1/2\mathcal{W}_0(\psi_i) & \text{if } i > i^*, \end{cases} \quad (10)$$

where i^* is the smallest value of i that satisfies $\phi_i < \phi_{i+1}$, i.e.,

$$i^* = \arg \min_i \psi_i.$$

Fig. 6 depicts the analytically obtained values of $\mathbb{E}\tau_i$ in a long network ($N = 20$) (10) for several values of the packet dropping probability ξ .

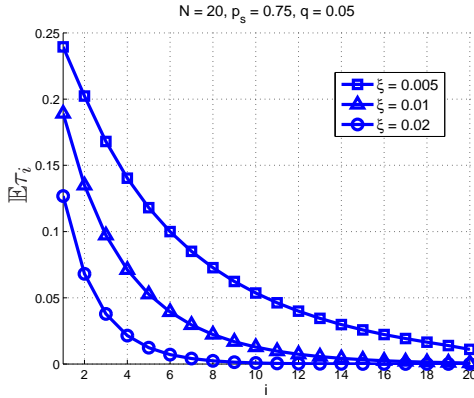


Fig. 6. Analytical approximation of the mean occupancies of nodes (9) in a long ($N = 20$) flow with $p_s = 0.75$ and $q = 0.05$.

End-to-end Delay, Throughput and Reliability: We now derive analytical expressions for the throughput, end-to-end delay and reliability in terms of the steady state node occupancies, for the general case ($R < 1$).

Proposition 3.2: For a (typical) ALOHA-based flow along N relay nodes, we have the following.

(a) The steady-state throughput is

$$T = qp_s \mathbb{E}\tau_N. \quad (11)$$

(b) The delay experienced by a packet at the i^{th} node, $0 \leq i \leq N$, follows a geometric distribution with parameter

$$s_i = qp_s(1 - \mathbb{E}\tau_{i+1}). \quad (12)$$

Consequently, the mean end-to-end delay is

$$D = \sum_{i=0}^N (qp_s(1 - \mathbb{E}\tau_{i+1}))^{-1}. \quad (13)$$

(c) The end-to-end reliability of the network is

$$R = \prod_{i=0}^N \frac{s_i(1 - \xi)}{s_i + \xi - s_i\xi}, \quad (14)$$

Proof: The proof of (a) is similar to the proof of Lemma 3.1. Indeed, as explained earlier, the probability that the packet at node N successfully hops to the destination in one time slot is $qp_s \mathbb{E}\tau_N$.

In order to prove (b), let us suppose that a packet arrives at an arbitrary node i , $0 \leq i \leq N$. The three events that need to occur in the following order for the packet to be able to hop to node $i + 1$ successfully are:

- (1) Node i transmits its packet.
- (2) Node $i + 1$ has an empty buffer.
- (3) Node i 's transmission is successful.

Since the node occupancies are assumed to be independent of each other (by the mean-field approximation), the probability of node $i + 1$ having an empty buffer conditioned on the fact that a packet arrives at node i is still $1 - \mathbb{E}\tau_{i+1}$. The events (1), (2) and (3) are also clearly independent of each other, thus the probability that it hops successfully to $i + 1$ in a time slot is

$$s_i = qp_s(1 - \mathbb{E}\tau_{i+1}).$$

Consequently, the delay experienced by a packet at node i is geometrically distributed with mean $1/s_i$.

We now derive (c), i.e., compute the fraction of packets successfully hopping from node i to $i + 1$, $1 \leq i \leq N$. Suppose the packet stays at node i for n_i slots before hopping to node $i + 1$. The reliability r_i across the link $i \rightarrow i + 1$, is

$$r_i = (1 - \xi)^{n_i} \quad (15)$$

From (12), we know that $n_i \sim \text{Geo}(s_i)$. Therefore, we get

$$r_i = \sum_{k=1}^{\infty} (1 - s_i)^{k-1} s_i (1 - \xi)^k = \frac{s_i(1 - \xi)}{1 - (1 - s_i)(1 - \xi)}.$$

The end-to-end reliability is simply $R = \prod_{i=0}^N r_i$, which is equivalent to (14). ■

Fig. 7 depicts the achievable throughput, mean end-to-end delay and reliability values for a typical flow in the considered ad hoc network model for $p_s = [0.1, 0.2, \dots, 0.9, 1]$, $q = 0.2$ and $N = 5$. The corresponding empirical values are also plotted (dashed lines), and are shown to closely match the analytical curves. We see that in the noise-limited regime, the average end-to-end delay may be reduced significantly by increasing the packet dropping probability. The tradeoff is that increasing ξ also results in emptying some buffers in the network, thus the reliability and throughput performances of the ad hoc network deteriorate.

B. The Interference-limited Regime

Typically, the performance of ad hoc networks is limited not only by thermal noise but also by the interference in the network due to concurrent transmissions. We argue that in order to study this general case, it is sufficient to analyze the case where the system is purely interference-limited. Indeed, under the conditions of Rayleigh fading, the success probability across a typical link in the PPP network is equal to the product of the Laplace transforms of noise and interference [11]. Since the Laplace transform of noise for any given value

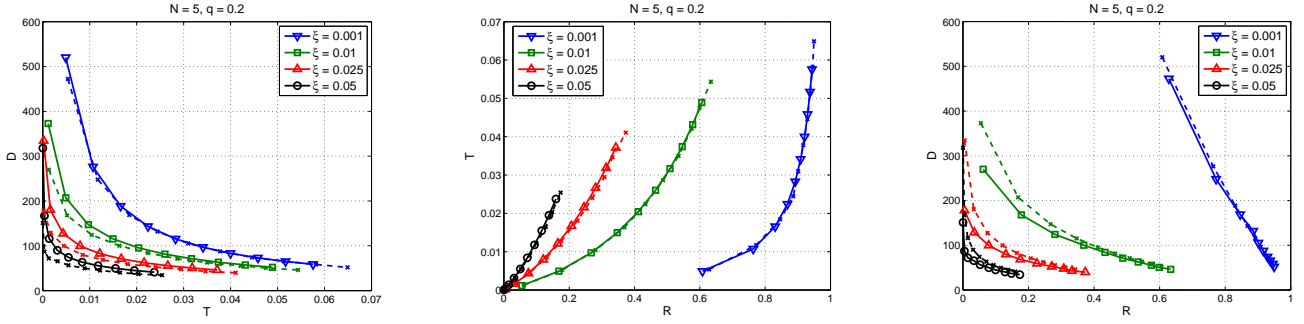


Fig. 7. Analytically (solid lines) and empirically (dashed lines) obtained TDR Tradeoffs for an ad hoc network flow along $N = 5$ relays. In the noise-limited regime, increasing ξ helps reduce the end-to-end delay significantly, although the throughput and reliability performances worsen.

of Θ is independent of the occupancies of other nodes in the network (or equivalently, of the packet dropping process), the effective value of the link reliability (in the general case) is simply the link reliability for the interference-limited case scaled down by a constant factor. Thus, it is adequate to analyze the TDR performance for the interference-limited regime, and the results extend directly for the general scenario. In this section, we define the success probability (across a typical link) as $p_s = \mathbb{P}(\text{SIR} \geq \Theta)$, which critically depends on the occupancies of other nodes in the network.

1) **Case 1: $R = 1$:** We first consider the case with 100% reliability, i.e., all packets are retransmitted until successfully received. Recall from Subsubsection III-A1 that when $R = 1$, the product of throughput and mean end-to-end delay is equal to $1 + N/2$ (as a consequence of Little's theorem). Thus, the TD curve is a hyperbola along the $R = 1$ axis (equivalent to the plot in Fig. 4).

2) **Case 2: $R < 1$:** Next, we consider the case where $R < 1$. Note that dropping a fraction of packets leads to a decreased intensity of interfering nodes in the network, thus the link reliabilities increase with increasing ξ . We now proceed to derive the success probability across a typical link.

To this end, suppose that the mean node occupancies for a typical flow at steady state are $(1, \mathbb{E}\tau_1, \dots, \mathbb{E}\tau_N)$. The average number of potential interferers in each flow is $1 + \sum_{i=1}^N \mathbb{E}\tau_i$. With δ being the density of source nodes (or flows) and q the ALOHA contention probability, it follows that the density of interferers is at most⁶

$$\lambda_I \lesssim \delta q \left(1 + \sum_{i=1}^N \mathbb{E}\tau_i \right). \quad (16)$$

Even though transmissions in the network are completely uncoordinated, the interference is actually spatially and temporally correlated owing to the presence of common randomness in the locations of nodes [12]. However, for analytical tractability, we make the relaxed assumption that the set of interfering nodes forms a PPP with density λ_I , which

⁶This term is actually an upper bound, owing to the existence of relay nodes having multiple packets in its buffer (corresponding to several flows). The bound is tight for small q (when the density of interferers is small), or small δ (when the flows in the network themselves are sparse).

is actually quite reasonable at small q [12]. We have the following lemma concerning the success probability across a typical link in the considered system model.

Lemma 3.3: For the ALOHA-based ad hoc network, the probability of a successful transmission $p_s = \mathbb{P}[\text{SIR} > \Theta]$ for a typical link is

$$p_s = \left(\frac{(1 - \delta)\phi}{(1 - \delta)\phi + 2\lambda_I c} \right)^n, \quad (17)$$

where λ_I is the intensity of interferers, and $c = \pi\Gamma(1 + 2/\gamma)\Gamma(1 - 2/\gamma)\Theta^{2/\gamma}$.

Proof: This is a generalization of [13, Prop. 5.1] ■

Substituting for λ_I in (17) using (16), we obtain the success probability across a typical link to be lower-bounded as

$$p_s \gtrsim \left(\frac{(1 - \delta)\phi}{(1 - \delta)\phi + 2\delta q \left(1 + \sum_{i=1}^N \mathbb{E}\tau_i \right) c} \right)^n, \quad (18)$$

where the approximation is tight for small q .

The steady state occupancies of nodes, $\mathbb{E}\tau_i$, $1 \leq i \leq N$, may be obtained by simultaneously solving the set of N non-linear equations (6), where the value of p_s is as given by (18).

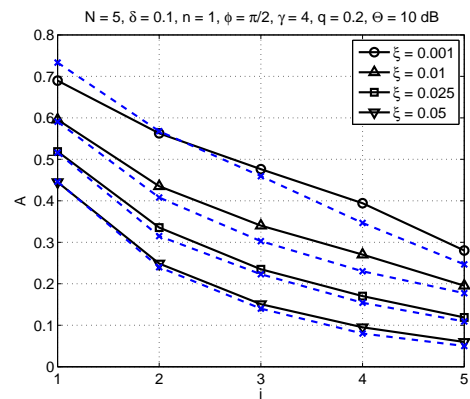


Fig. 8. Values of $\mathbb{E}\tau_i$ obtained numerically (solid lines) using (6) for some system parameter values. The empirical values (dashed lines) are also plotted, and are seen to match the theoretical ones closely.

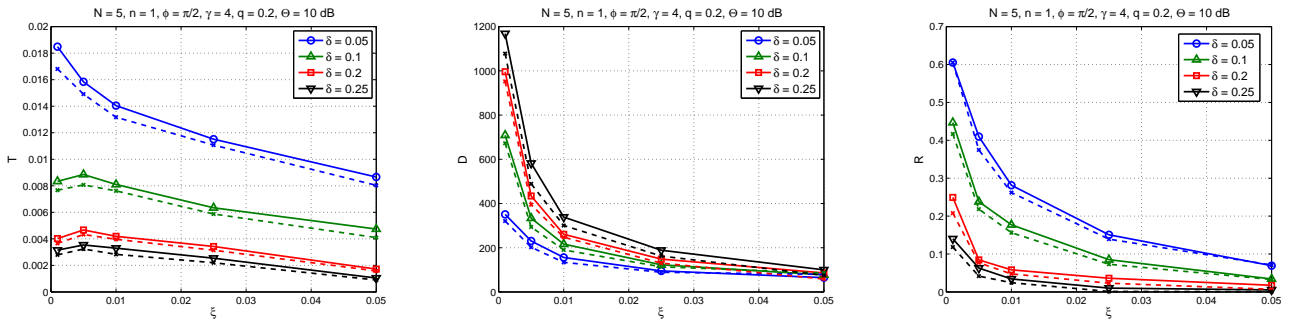


Fig. 9. TDR performances of the ALOHA-based flow versus ξ , for $N = 5$, $n = 1$, $\phi = \pi/2$, $\gamma = 4$, $q = 0.2$ and $\Theta = 10$ dB. The empirical results (dashed lines) match the analytical ones (solid lines). Note that at high δ , dropping a small fraction of packets can actually help improve the system throughput.

Fig. 8 shows numerically obtained values (solid lines) of the mean node occupancies for $N = 5$, $n = 1$, $\phi = \pi/2$, $\gamma = 4$, $q = 0.2$ and $\Theta = 10$ dB, and several values of ξ . The corresponding empirical values (dashed lines) are also plotted, and they corroborate the values obtained numerically.

The throughput, delay and reliability performances of the multihop flow are quantified using (11), (13) and (14) respectively, together with values of the mean node occupancies. Fig. 9 depicts the TDR performances of the ALOHA-based line network versus ξ , in the interference-limited regime, for some values of the system parameters. We observe the following.

- When δ is small (for example, when $\delta = 0.05$), increasing the packet dropping probability ξ reduces the system throughput. However, as δ gets larger (for instance, when $\delta = 0.1$), dropping a few packets helps mitigate the interference, thus the link reliabilities increase, and the throughput across a typical flow improves (for example, at $\xi = 0.005$). As ξ increases further, the loss in throughput due to dropped packets exceeds the gain in throughput due to interference mitigation, and the throughput begins to fall. Indeed, there exists an optimum value of ξ that maximizes the throughput of the flow.
- As expected, with increasing ξ or decreasing δ , the mean end-to-end delay decreases; the reliability also suffers.

IV. SUMMARY

We considered a multihop wireless network consisting of several source-destination pairs communicating with each other in a completely uncoordinated manner. Employing the mean-field approximation, we presented a framework for computing the steady state mean node occupancies, and quantifying the network's TDR performance. As intuitively expected, we found that in the noise-limited regime, dropping a small fraction of packets in the network leads to a smaller end-to-end delay at the cost of reduced throughput, whereas, in the interference-limited scenario, dropping a few packets in the network can help mitigate the interference in the network leading to an increased throughput. We also provided some empirical (simulation-based) results to corroborate the theory.

In conclusion, we view this work as an initial effort towards understanding the throughput, delay and reliability tradeoffs in

multihop wireless networks. Extending the analysis in order to accommodate different source traffic models such as constant bit rate and Bernoulli, other MAC schemes such as CSMA and spatial TDMA, and more sophisticated packet dropping strategies such as those based on bounded delay and limited retransmissions are interesting directions for future work.

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