Path Loss Exponent Estimation in a Large Field of Interferers

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Motivation

- Large-scale path loss law: $S \propto d^{-\gamma}$.
- Though it is typically assumed that the path loss exponent (PLE) is known a priori, it is often not the case.
- The PLE has a strong impact on the quality of links, and thus needs to be accurately estimated.

Motivation (contd.)

- **Example 1:** The information-theoretic capacity of large random ad hoc networks scales as *

$$\begin{aligned} n^{2-\gamma/2} & \quad \text{for} \quad 2 \leqslant \gamma < 3 \\ \sqrt{n} & \quad \text{for} \quad \gamma \geqslant 3. \end{aligned}$$

Depending on the value of γ, different routing strategies are required to be implemented.

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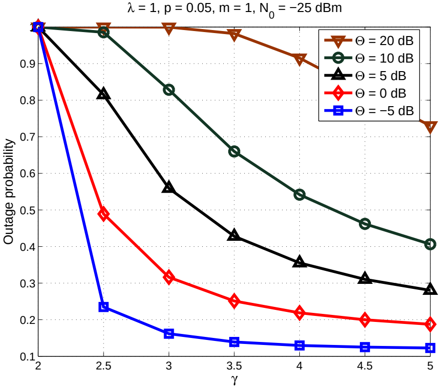
*A. Özgür, O. Lévêque and D. Tse, "Hierarchical Cooperation Achieves Optimal Capacity Scaling in Ad Hoc Networks," *IEEE Trans. Info. Th.*, 2007.

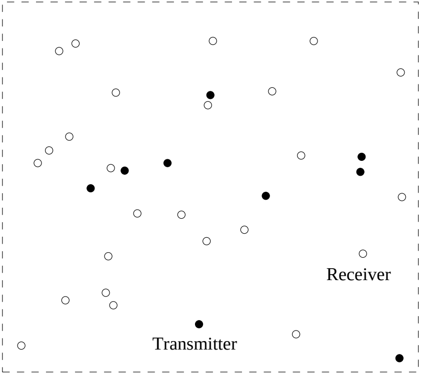
Motivation (contd.)

- **Example 2:** Outage probability in a planar Poisson point process with Rayleigh fading.



The system performance critically depends on γ.

System Model



Filled circles: transmitters.
Empty circles: receivers.

- Poisson point process (PPP) on $\mathbb{R}^2$ with density λ.
- Channel access scheme is ALOHA.
- p is the ALOHA contention parameter.
- No synchronization.

System Model (contd.)

- Attenuation in the channel: product of
  - large-scale path loss, with PLE γ.
  - small-scale fading (m-Nakagami).
    $m = 1$: Rayleigh fading ; $m \to \infty$: no fading.
- Noise is AWGN with variance N_0.
- All the transmit powers are equal to unity (no power control).

Problem: How do you accurately estimate the PLE at each node in the network in a completely distributed manner?

What Makes Estimating the PLE Complicated?

- Large-scale loss: **deterministic**; small-scale fading: **stochastic**.
- This distinction does not hold in random networks, thus need to consider the **distance and fading ambiguities jointly**.
- During the network initialization phase of the network, the system is typically **interference-limited**.

Purely RSS-based estimators cannot be used in these situations.

Overview

- Propose three distributed algorithms for estimating the PLE in large random wireless networks that explicitly take into account
  - the uncertainty in the locations of the nodes.
  - the uncertainty in the fading gains across links.
  - the interference in the network.
- Provide simulation results to demonstrate the performance of the algorithms and quantify the estimation errors.
- Metric: 'relative' MSE: $\mathbb{E}\left[(\hat{\gamma}-\gamma)^2\right]/\gamma$.

Algo. 1: Using the Mean Interference

- This algorithm assumes that the network density λ is known.
- In theory, the mean interference is

$$\mu = 2\pi\lambda p A_0^{2-\gamma} / (\gamma - 2) \,,$$

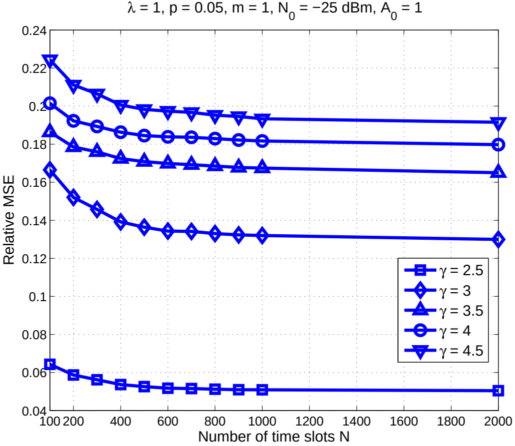
where A_0 is the near-field radius of the antenna.

Implementation

- Nodes simply need to record the mean strength of the received power, μ', averaged over several time slots.
- Equating μ to μ', and using the known values of p, A_0, and λ, $\hat{\gamma}$ is found from a look-up table.

Algo. 1: Using the Mean Interference (contd.)

Relative MSE of $\hat{\gamma}$ versus the number of time slots.



The estimates are fairly accurate over a wide range of parameters.

Algo 2: Based on (Virtual) Outage Probabilities

- This algorithm does not require the knowledge of λ or m.
- In a Poisson network, the outage probability is

$$\Pr(\mathcal{O}) = \mathbb{P}(\mathsf{SIR} \leqslant \Theta) = 1 - \exp(-c\Theta^{2/\gamma}),$$

  where $c = \lambda p \pi \Gamma\left(m + \frac{2}{\gamma}\right)\Gamma\left(1 - \frac{2}{\gamma}\right) \Big/ \left(\Gamma(m) m^{2/\gamma}\right)$.
- Nodes can determine the SIR (and consequently, the outage probability) by considering a 'virtual' transmitter.

Algo 2: Based on (Virtual) Outage Prob. (contd.)

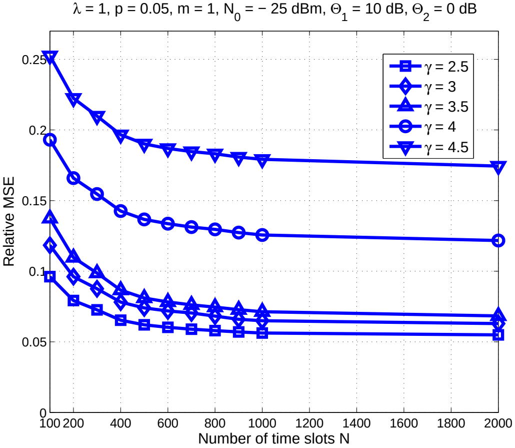
Implementation: A 'differential' method.

- Obtain a histogram of the observed SIR values measured over several time slots.
- The empirical success probabilities ($p_{s,i} = \mathbb{P}(\mathsf{SIR} > \Theta_i)$, $i = 1, 2$) are obtained at two different threshold values.
- An estimate of γ is obtained as

$$\hat{\gamma} = \frac{2 \ln(\Theta_1 / \Theta_2)}{\ln\left(\ln p_{s,1} / \ln p_{s,2}\right)}.$$

Algo 2: Based on (Virtual) Outage Prob. (contd.)

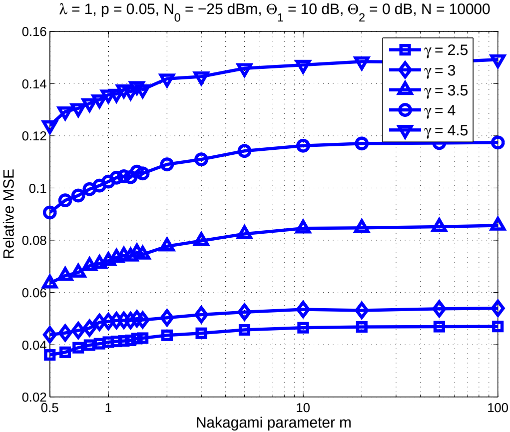
Relative MSE of $\hat{\gamma}$ versus the number of time slots.



The estimation error increases with larger γ.

Algo 2: Based on (Virtual) Outage Prob. (contd.)

Relative MSE of $\hat{\gamma}$ versus the Nakagami parameter.



This algorithm performs more accurately at lower values of m.

Algo 3: Based on the Cardinality of the Tx Set

- This algorithm also does not require to know m or λ.
- Transmitter node y is in receiver node x's *transmitting set*, $\mathsf{T}(x)$ if they are connected, i.e., the SIR at x due to y's signal is $> \Theta$.
- For $m \in \mathbb{N}$,

$$\mathbb{E}|\mathsf{T}(x)| = \bar{N}_{\mathsf{T}} = \frac{\Gamma(m)\left(1 - \left(\frac{2}{\gamma}\right)^{m}\right)}{\Gamma(m + \frac{2}{\gamma})\Gamma(2 - \frac{2}{\gamma})\Theta^{2/\gamma}}.$$

Algo 3: Based on the Card. of the Tx Set (contd.)

Implementation

- For a known threshold $\Theta_1 \geqslant 1$, at time slot i, $1 \leqslant i \leqslant N$, set

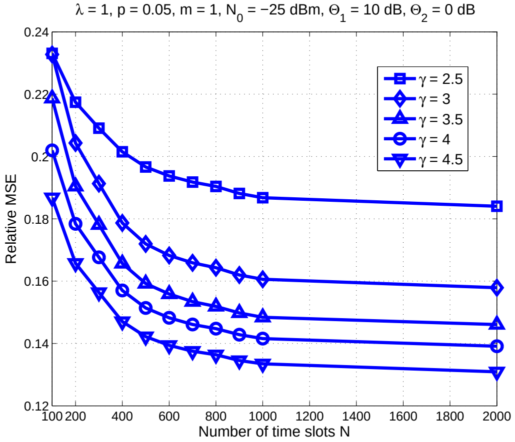
$$N_{T,1}(i) = \begin{cases} 1 & \text{if the node can decode a packet} \\ 0 & \text{otherwise.} \end{cases}$$

- Evaluate $\bar{N}_{T,1}$ and $\bar{N}_{T,2}$ at two different threshold values Θ_1 and Θ_2 respectively.
- We obtain

$$\hat{\gamma} = \left(2 \ln(\Theta_2/\Theta_1)\right) / \ln(\bar{N}_{T,1}/\bar{N}_{T,2}).$$

Algo 3: Based on the Card. of the Tx Set (contd.)

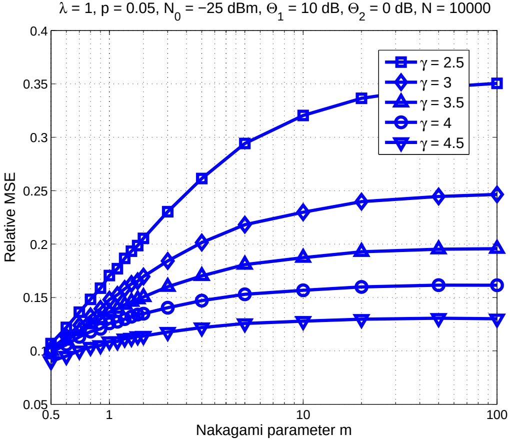
Relative MSE of $\hat{\gamma}$ versus the number of time slots.



In contrast to Algo. 1 and 2, the relative MSE decreases with γ.

Algo 3: Based on the Card. of the Tx Set (contd.)

Relative MSE of $\hat{\gamma}$ versus the Nakagami parameter.



The estimates are more accurate at lower m.

Summary

- We have addressed the PLE estimation problem in the presence of node location uncertainties, m-Nakagami fading and most importantly, interference!
- Each of the algorithms are fully distributed and do not require any information on the location of other nodes or the value of m.
- We remark that the bias (and the MSE) can be significantly lowered if nodes have access to several independent realizations of the point process or if they are allowed to communicate.