Combining Stochastic Geometry and Statistical Mechanics for the Analysis and Design of MANETs

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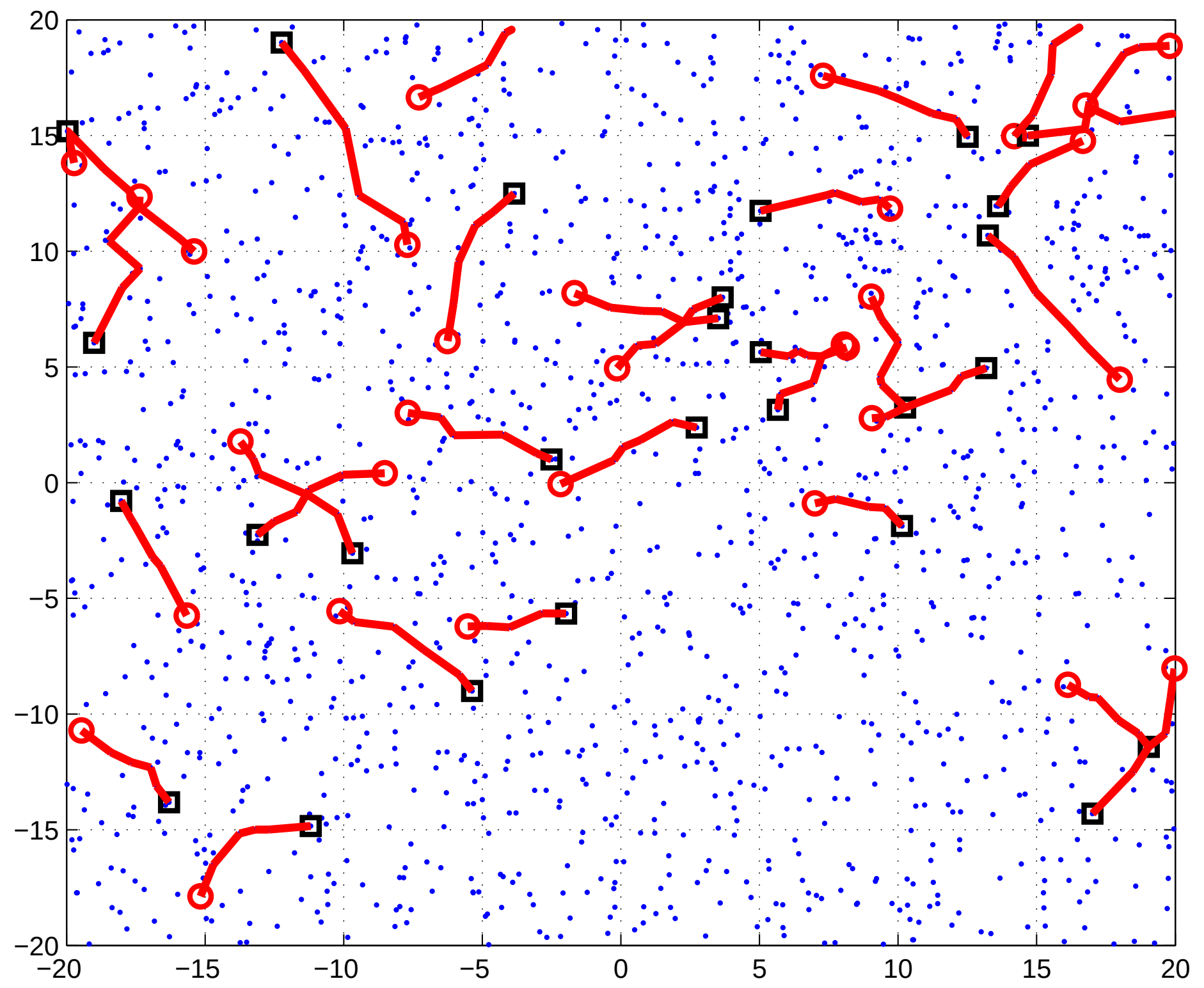
OVERVIEW

- Employ a combination of tools from two paradigms, **stochastic geometry** and **TASEP** (Totally Asymmetric Simple Exclusion Process) to analyze multihop wireless ad hoc networks.
- Provide valuable insights from a system design stand-point.

Prior work: single-hop analysis, backlogged nodes, simplified models.

SYSTEM MODEL



Sources $\sim$ PPP (λ_s) $\quad (= \delta\lambda)$
Potential relays $\sim$ PPP (λ_r) $\quad (= (1-\delta)\lambda)$.
Channel: Rayleigh i.i.d., path loss exponent γ.

Routing Strategy:

In each flow, the next-hop node is the n^{th}-nearest-neighbor that lies within $\pm\phi/2$ of the axis to the destination.

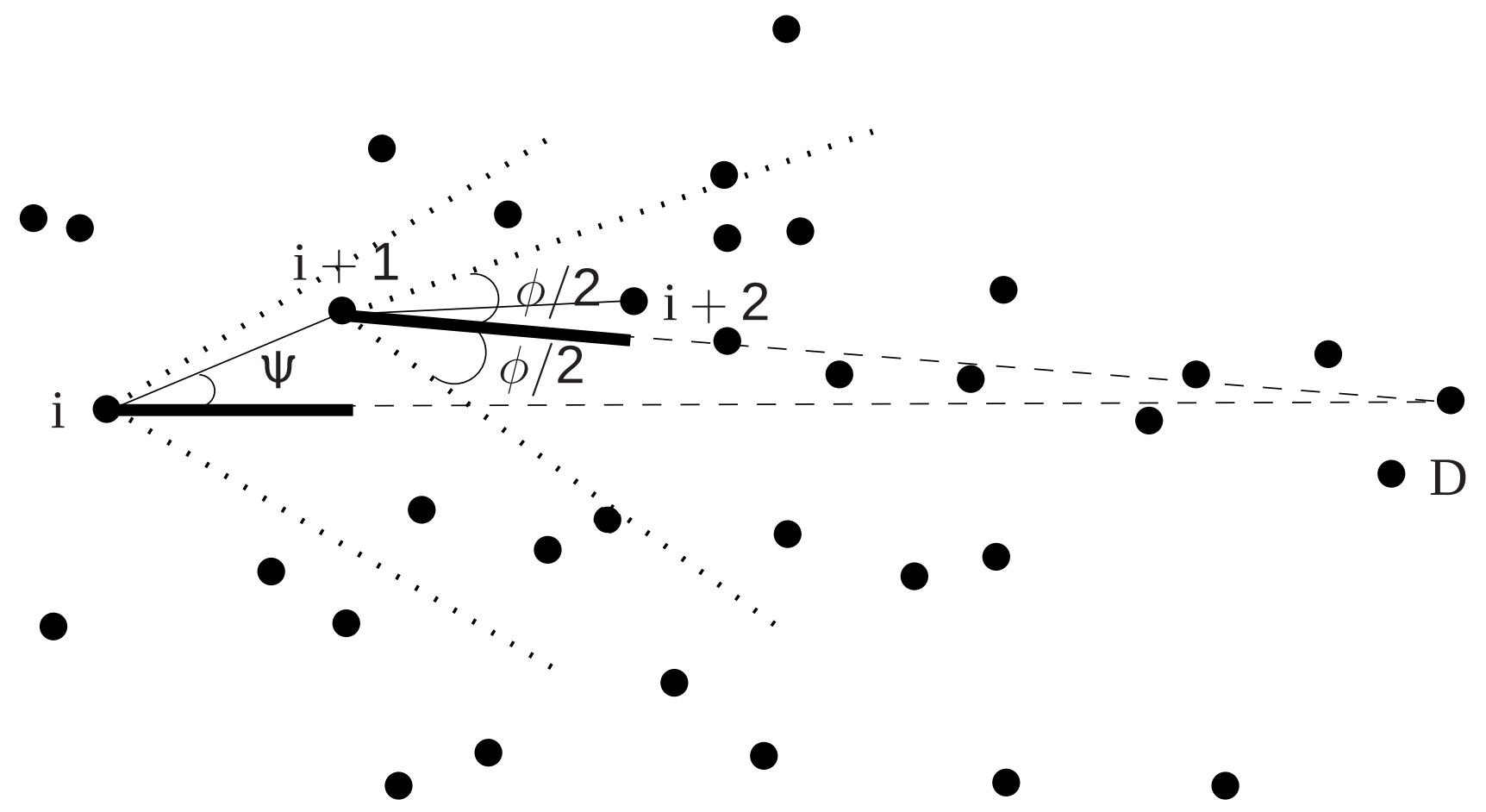


Illustration of nearest-neighbor ($n = 1$) routing.

MAC Schemes:

- **Randomized TDMA (r-TDMA)**: The transmitting node in each time slot is chosen uniformly randomly from the set of all nodes in the flow *having a packet*.
- **ALOHA** with contention probability q.

Performance Metric:

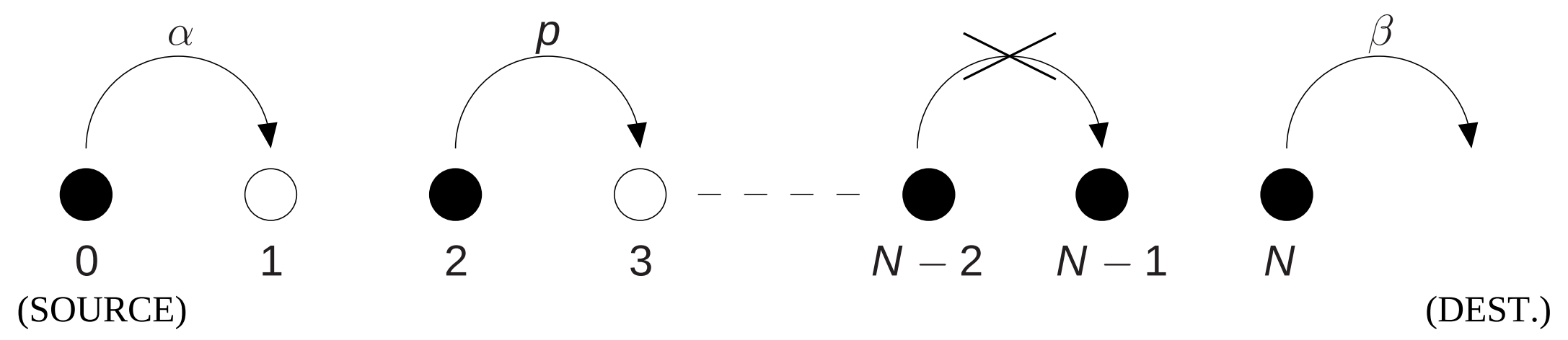
Transport Density is the average number of bit-meters successfully delivered per second per unit surface area: $\rho_{\text{trans}} = \delta T D$, where
T is the throughput of a 'typical' flow.
D denotes the progress of packets from source to destination.

THE TOTALLY ASYMMETRIC SIMPLE EXCLUSION PROCESS

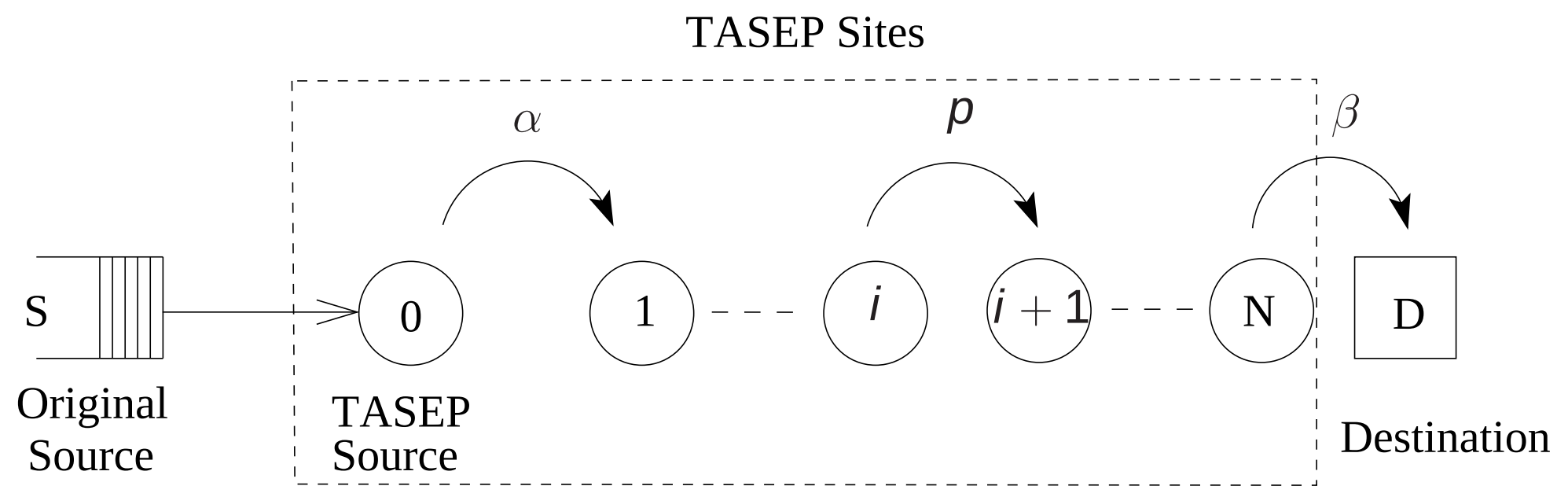
Transmission Policy:

1. All the buffering in the network is performed at source nodes.
2. Transmissions are not attempted if the adjacent relay's buffer already contains a packet.
3. Packets are retransmitted until successfully received.

TASEP Model:



Network flows:



ANALYSIS OF R-TDMA-BASED WIRELESS NETWORKS

Proposition 1 *For the r-TDMA-based ad hoc network, the probability of a successful transmission* $p_s = \mathbb{P}[SIR > \beta]$ *from any node to its* n^{th}*-nearest-neighbor in a sector* ϕ *is*

$$p_s = \left(\frac{(1-\delta)\phi}{(1-\delta)\phi + 2\delta c}\right)^n,$$

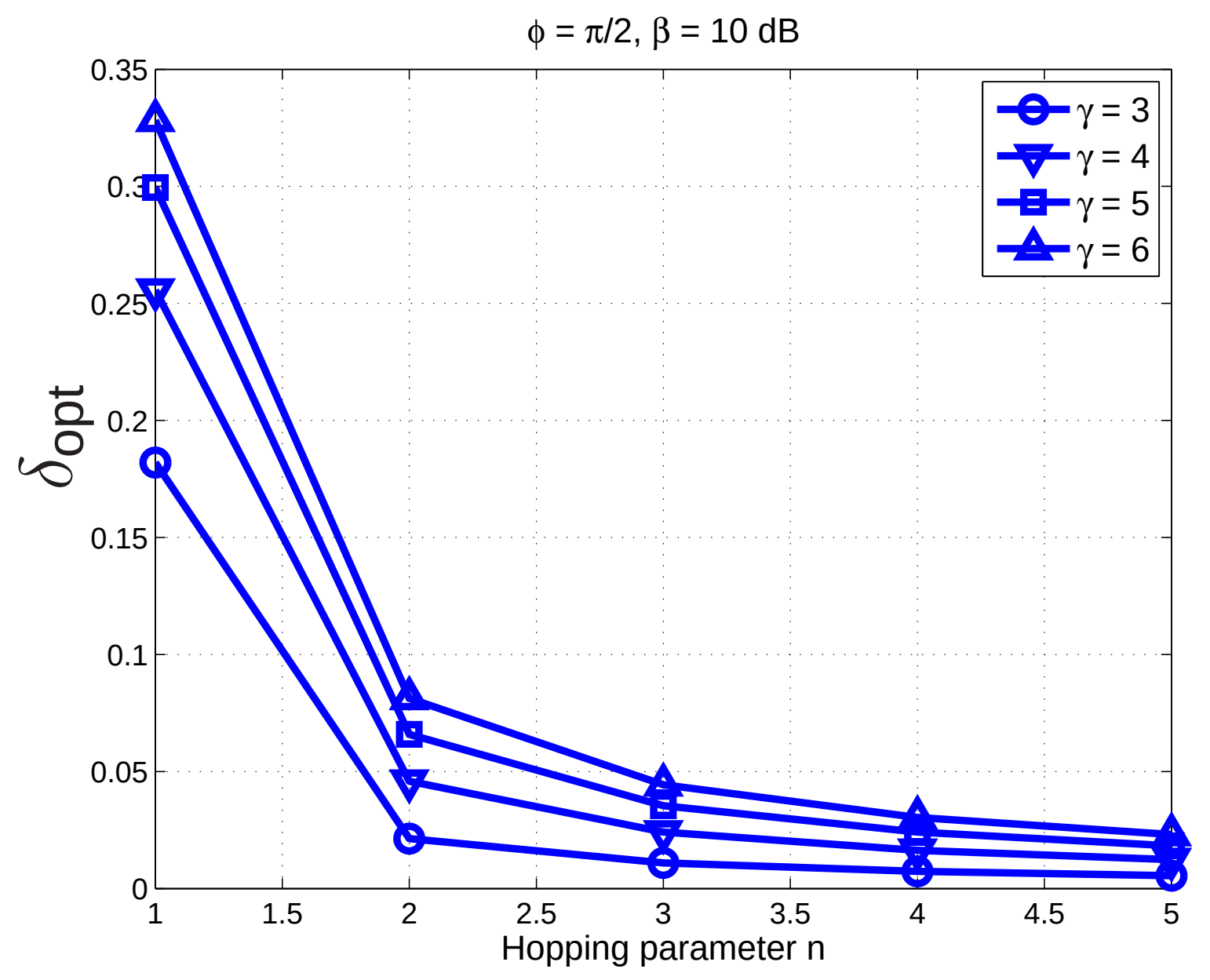
where $c = \pi\Gamma(1+2/\gamma)\Gamma(1-2/\gamma)\beta^{2/\gamma} = \frac{2\pi^2\beta^{2/\gamma}}{\gamma\sin(2\pi/\gamma)}$.

Proposition 2 *For the r-TDMA-based flow with N relays and success probability* p_s*, the throughput at steady state is*

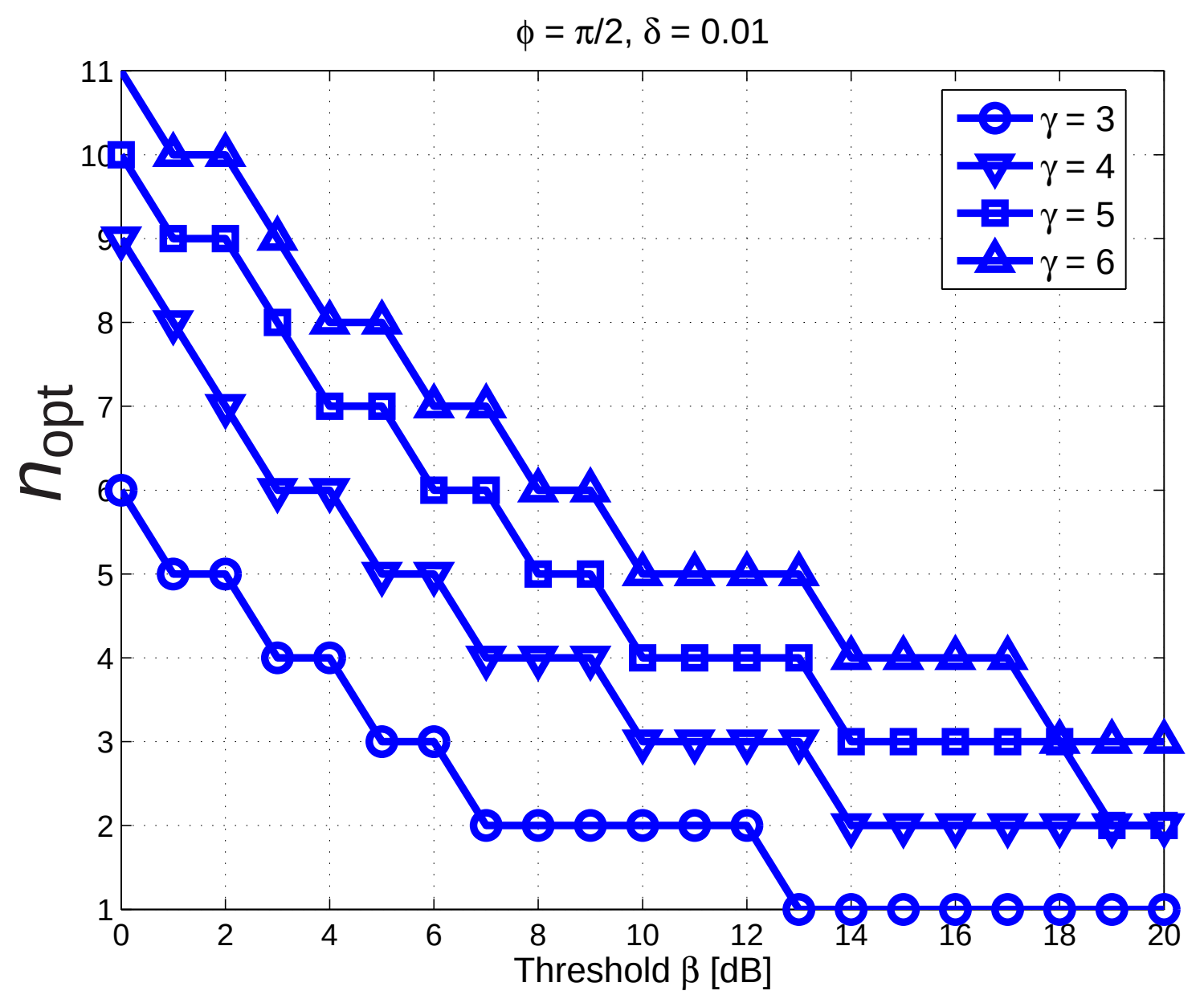
$$T = \frac{p_s}{2N+1}.$$

Using the definition of transport density, we obtain

$$\boxed{\rho_{\text{trans}} = \frac{N}{(2N+\sqrt{n})}\frac{\delta(1-\delta)^{n-\frac{1}{2}}\phi^{n-\frac{3}{2}}}{((1-\delta)\phi+2\delta c)^n}\sqrt{2\pi n}\sin\left(\frac{\phi}{2}\right).}$$



$$\delta_{\text{opt}} = \frac{3\phi+4(n-1)c-\sqrt{(\phi+4(n-1)c)^2+16n\phi c}}{2(\phi-2c)}.$$



$$n_{\text{opt}} \approx \left\lceil 0.5 / \ln\left(1 + \frac{2\delta c}{(1-\delta)\phi}\right)\right\rceil.$$

ANALYSIS OF ALOHA-BASED WIRELESS NETWORKS

For analytical tractability, we assume the following.

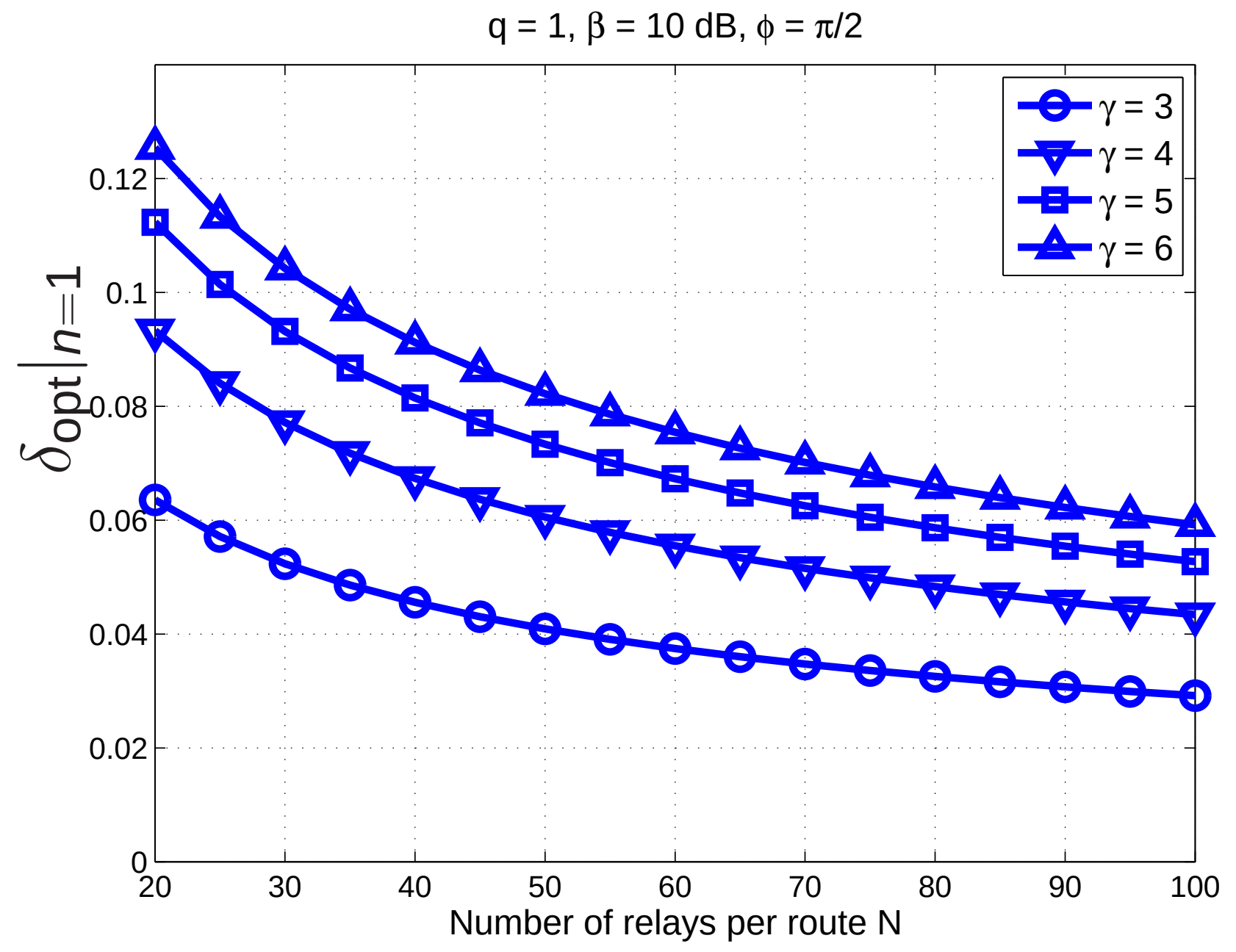
- We consider long flows, i.e., take $N \gg 1$.
- We employ the *mean field theory* assumption.

Proposition 3 *For the ALOHA-based network with N relay nodes, the packet success probability* p_s *from any node to its* n^{th}*-nearest-neighbor is*

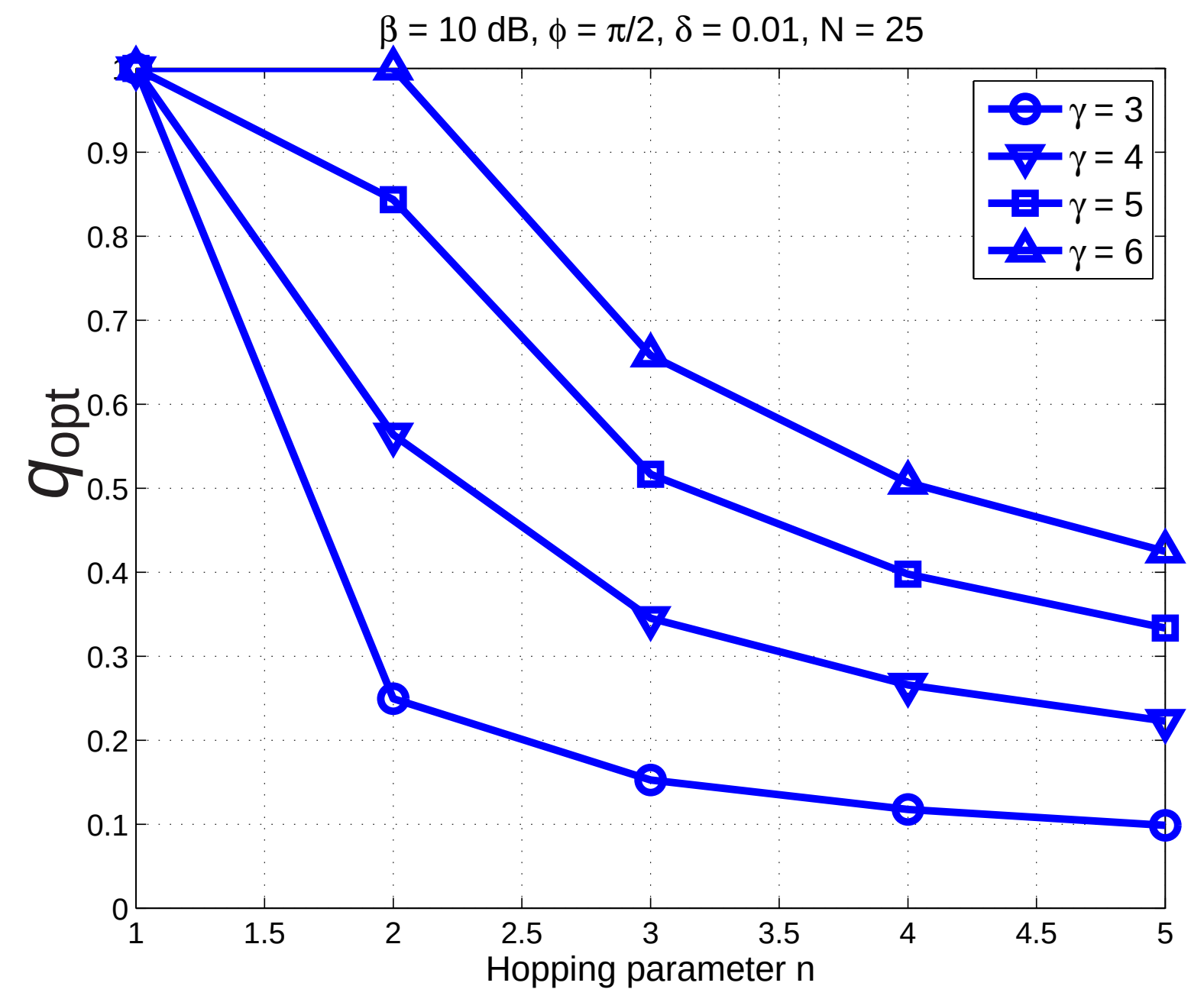
$$p_s \gtrapprox \left(\frac{(1-\delta)\phi}{(1-\delta)\phi + \delta qNc}\right)^n.$$

The transport density may be lower-bounded as

$$\boxed{\rho_{\text{trans}} \gtrapprox \frac{Nq\delta(1-\delta)^{n-\frac{1}{2}}\phi^{n-\frac{3}{2}}n^{n/2}}{((1-\delta)\phi\sqrt{n}+\delta qNc)^n}\sqrt{\frac{\pi}{8}}\sin\left(\frac{\phi}{2}\right).}$$

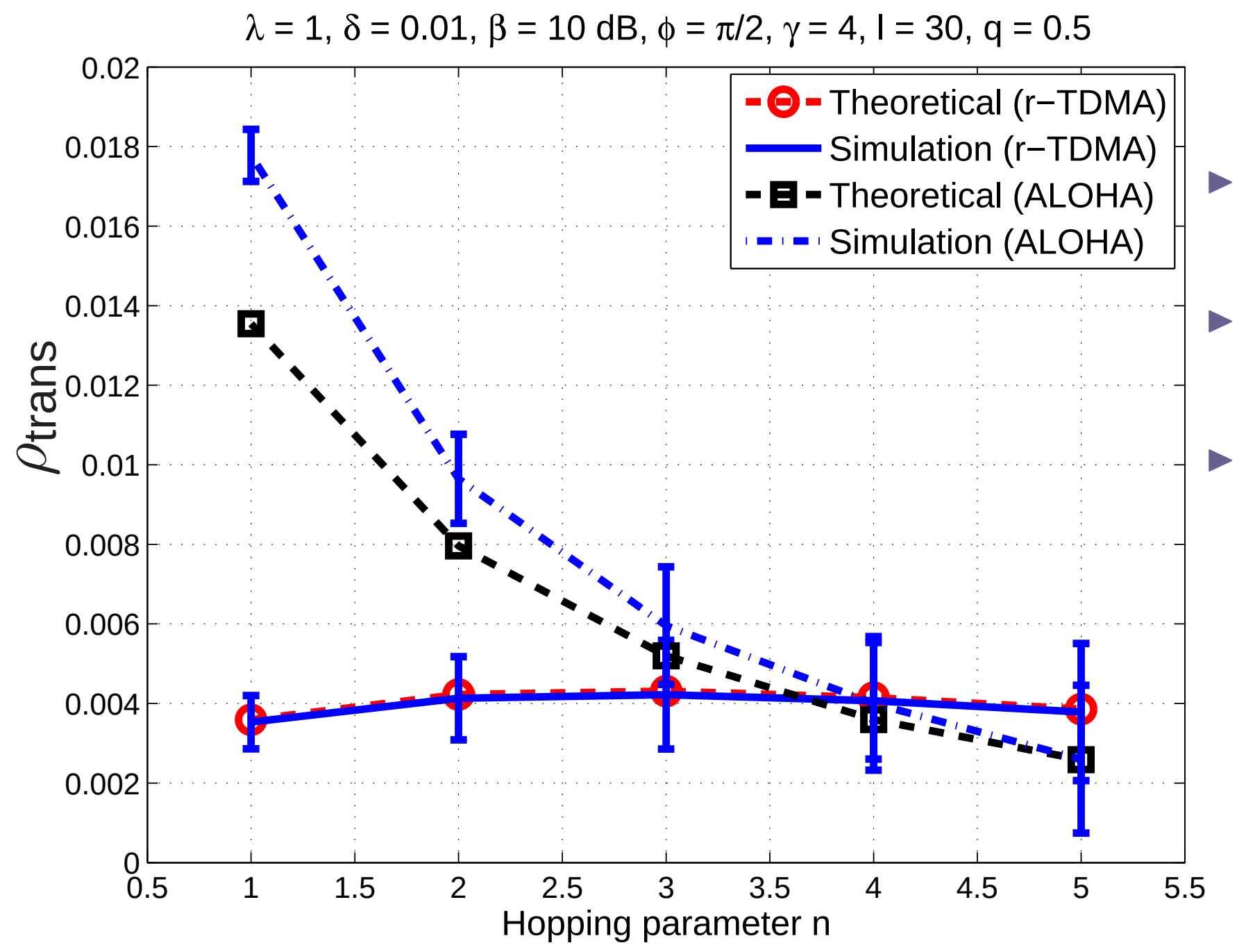


$$\delta_{\text{opt}}|_{n=1} = \frac{3\phi-\sqrt{\phi^2+8\phi Nc}}{2(\phi-Nc)}.$$



$$q_{\text{opt}} = \min\left\{1, \frac{(1-\delta)\phi\sqrt{n}}{\delta Nc(n-1)}\right\}.$$

SIMULATION RESULTS



- Verification needed since relays may serve multiple nodes.
- Based on 100 different realizations of the point process on a 40×40 square.
- There exist regimes where either scheme performs better.

FUTURE WORK

- Propose distributed routing algorithms for MANETs that are 'optimal' in nature.
- Characterize the performance of other MAC schemes, in particular CSMA and spatial TDMA.
- Use ideas from *Langmuir Kinetics* to characterize the TDR region in wireless ad hoc networks.