New Paradigms for the Analysis of Ad Hoc Networks

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Oral Candidacy Exam
Tuesday, March 31, 2009

Introduction and Motivation

- The past few years have witnessed several revolutionary research developments in wireless communications and networking.
- However, inherent roadblocks have stunted our progress from the era of tetherless connectivity to the era of ubiquitous connectivity [†] .
  - Classical information theory is inadequate to study ad hoc networks.
  - Correlations between flows across various links need to be explicitly considered during analysis and design.
  - An adaptive cross-layer design approach needs to be implemented.
- Consequently, we need to consider other subject areas to obtain ideas and methodologies.

[†] J. Andrews, N. Jindal, M. Haenggi, R. Berry, D. Guo, M. Neely, S. Weber, S. Jafar and A. Yener, "Rethinking information theory for mobile ad hoc networks," *IEEE Communications Magazine*, Dec. 2008.

Outline

This proposal introduces two new paradigms for ad hoc network analysis.

Part 1: Distance Distributions

- Determine the distribution of internode distances in a finite uniformly random network using concepts from stochastic geometry.
- Study single-hop characteristics: Energy Consumption, Outage, Connectivity.

Part 2: The Totally Asymmetric Simple Exclusion Process (TASEP)

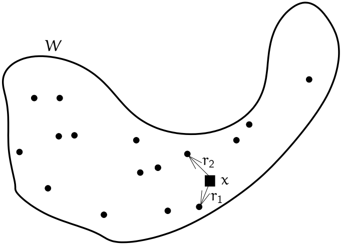
- Propose a distributed transmission policy for regulation of packet flow in multi-hop line networks.
- Analytically characterize the network end-to-end delay and throughput performance using tools from statistical mechanics.

Part 3: Extensions and Future Work

Part 1: Distance Distributions

Distances in Wireless Networks

- In wireless channels, received signal strength falls off with distance l as $l^{-\gamma}$, where γ is the path loss exponent (PLE).
- Thus, internode distances strongly impact the SINRs, and in turn the link reliabilities.
- Their knowledge is essential for the performance analysis and the design of efficient protocols and algorithms.
- **Question**: What are the statistics of the distance r.v. R_n from the reference point x to n^{th} nearest node of the point process?



A network with randomly deployed nodes on a set W.

Prior Work:

- Deals only with the first and perhaps second moments of distances.
- Characterizes the exact pdf only for simplified system models.

Our Contributions:

- Consider a realistic network model wherein a known number of nodes are randomly scattered over an area or volume W.
  - The nodal distribution follows a binomial point process (BPP).
- Derive the pdf and moments of the internode distances in a BPP with $W = b_d(o, R)$, i.e., a d-dimensional ball of radius R centered at the origin o.
- Apply findings to study single-hop characteristics of wireless networks such as energy consumption, outage and connectivity.

Internode Distance Distributions

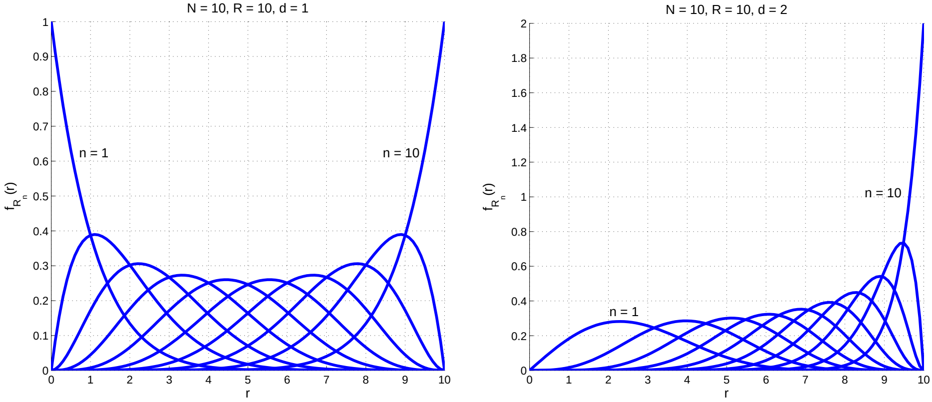
Theorem

In a BPP consisting of N *points uniformly randomly distributed in* $b_d(o, R)$*, the Euclidean distance* R_n *from the origin to its* n^{th} *nearest point follows a generalized beta distribution, i.e.,*

$$f_{R_n}(r) = \frac{d}{R}\frac{B(n - 1/d + 1, N - n + 1)}{B(N - n + 1, n)}\beta\left(\left(\frac{r}{R}\right)^d; n - \frac{1}{d} + 1, N - n + 1\right),$$

where $\beta(x; a, b)$ *is the beta density function defined as* $\beta(x; a, b) = \frac{1}{B(a,b)}x^{a-1}(1 - x)^{b-1}$ *and* $B(a, b)$*, the beta function:* $B(a, b) = \Gamma(a)\Gamma(b)/\Gamma(a + b)$.

Internode Distance Distributions (Contd.)



Distance pdfs in one- and two- dimensional BPPs.

Moments of the Internode Distances

- The Δ^{th} moment of R_n is ($\Delta \in \mathbb{R}$)

$$\mathbb{E}R_n^{\Delta} = \begin{cases} \frac{R^{\Delta}\Gamma(N+1)\Gamma(\Delta/d+n)}{\Gamma(n)\Gamma(\Delta/d+N+1)} & \text{if } n+\Delta/d > 0 \\ \infty & \text{otherwise} \end{cases}$$

- For $d = 1$, points are arranged as on a regular lattice on average. As $d \to \infty$, $\mathbb{E}R_n \to R$.
- For general d, $\mathbb{E}R_n^{\Delta}$ scales as $n^{\Delta/d}$.

Wireless Network Model

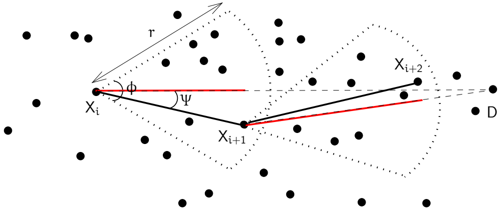
- N nodes are uniformly distributed in $b_d(o, R)$.
- Each node communicates with a base station (BS) located at o.
- Channel attenuation: Large scale path loss function g.
  - $g(x) = \|x\|^{-\gamma}$
- This model applies to cellular telephony networks and sensor networks.

Application 1: Energy Consumption

- The energy required to successfully deliver a packet over a distance l is l^γ.
- Average energy required to deliver packet from n^{th} nearest neighbor to the BS is γ-th moment of R_n.
  - Approximately scales as $n^{\gamma/d}$.
  - When $\gamma < d$, it is more energy-efficient to use longer hops, due to the sublinear behavior of $n^{\gamma/d}$.

Application 2: Design of Routing Algorithms

- Consider $d = 2$. Each node has a peak power constraint of $P \ll R^\gamma$ $\Rightarrow$ the communication range of each node is $r = P^{1/\gamma}$.
- Packets need to be forwarded from BS to a destination node D.
- Nodes adopt a greedy forwarding strategy.
  - Each node X_i that gets a packet relays it to its farthest neighbor in a sector of angle ϕ, $0 \leqslant \phi \leqslant \pi$, i.e., along $\pm\phi/2$ around the X_i-D axis.
  - The progress of the packet (marked in red) is defined as the effective distance travelled along the X_i-D axis.



Application 2: Design of Routing Algorithms (Contd.)

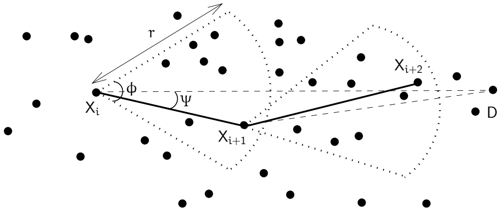
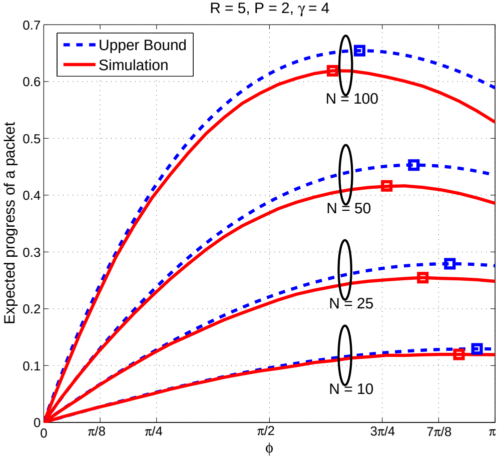


Illustration of the greedy forwarding strategy.

- Note the tradeoffs between ϕ and the progress of a packet.
  - Large ϕ: the direction of the farthest neighbor may be off the X_i-D axis.
  - Small ϕ: there are fewer nodes inside the sector.
- **Question**: What value of ϕ maximizes the expected progress of packets towards the destination?
- A simple upper bound can be derived by using knowledge of distance to farthest (N^{th}) neighbor.

Application 2: Design of Routing Algorithms (Contd.)



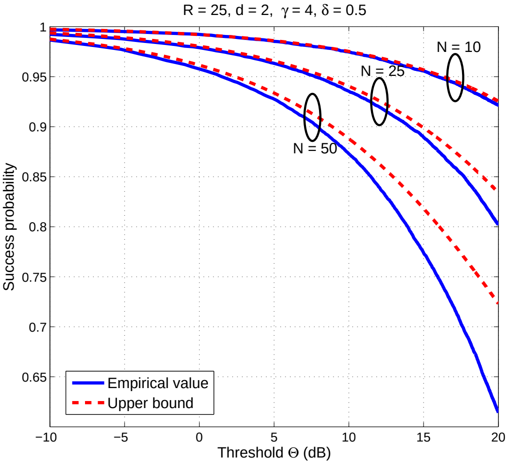
ϕ_{opt} gets smaller as N increases.

Application 3: Outage Probability

- Assume system is interference-limited, and received signal (from desired transmitter) power to be unity.
- MAC scheme: slotted ALOHA with contention parameter δ.
- An outage $\mathcal{O}$ is defined to occur when the SIR at the receiver is lower than a threshold Θ.
- The probability of outage is $\Pr(\mathcal{O}) = \Pr(I > 1/\Theta)$.
- Considering only the interference contribution from the nearest neighbor, we obtain

$$\Pr(\mathcal{O}) \geqslant \begin{cases} \delta \left(1 - \left(1 - \frac{\Theta^{d/\gamma}}{R^d}\right)^N\right) & \Theta \leqslant R^\gamma \\ \delta & \Theta > R^\gamma. \end{cases}$$

Application 3: Outage Probability (Contd.)



The nearest neighbor contributes to most of the interference.

Application 4: Connectivity

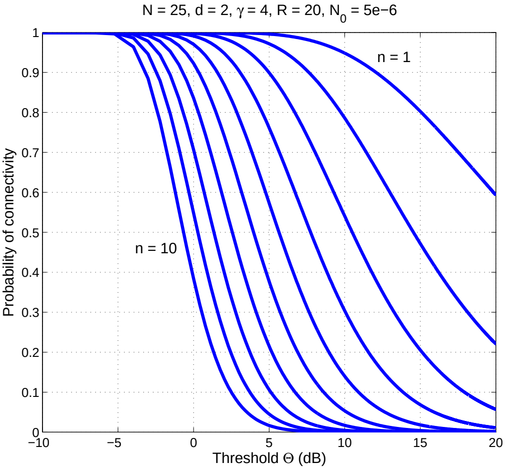
- Assume that interference can be controlled such that system is noise-limited.
- A node is connected to the BS if the SNR at the BS is greater than a threshold Θ.

$$\mathbb{P}_n = \begin{cases} 1 - I_{1-p'}(N-n+1, n) & \Theta > R^{-\gamma}/N_0 \\ 1 & \Theta \leqslant R^{-\gamma}/N_0, \end{cases}$$

where I is the incomplete beta function and $p' = \left((N_0\Theta)^{-1/\gamma}/R\right)^d$.

- The mean number of nodes that are connected to the BS is $N \min\{1, \left(\frac{(N_0\Theta)^{-1/\gamma}}{R}\right)^d\}$.

Application 4: Connectivity (Contd.)



Connection probability in a BPP with 25 nodes.

Part 2: The Totally Asymmetric Simple Exclusion Process (TASEP)

Multi-hop Line Networks

- Due to the stringent energy constraint in nodes and interference, a natural communication strategy is to reduce range of transmission.
- Multi-hop networks are not just meant to carry small volumes of data, but may also be intended for broadband services, e.g. mesh networks.
- However, existing buffering policies have inherent drawbacks: large queueing delays, non-coordinated transmissions, buffer overflows [†] .
- Consequently, the end-to-end delay and throughput performance in such systems is disappointing.

[†] Z. Fu, P. Zerfos, H. Luo, S. Lu, L. Zhang and M. Gerla, "The impact of multihop wireless channel on TCP throughput and loss," *INFOCOM 2003*.

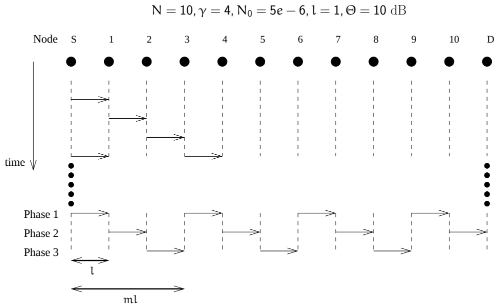
Prior Work:

- Focuses primarily on mean delays, delay correlations are mostly neglected.
- Considers very small or infinite networks.
- Neglects queueing delays/ assumes infinite buffer capacities/ considers backlogged nodes.

Our Contributions:

- Propose a revised transmission scheme to overcome the shortcomings.
- Draw analogies between packet flow in the multi-hop line network and the Totally Asymmetric Simple Exclusion Process (TASEP).
- Characterize end-to-end delay and throughput performance of the line network: first step to characterizing general ad hoc networks.

Optimality of ALOHA: The Sufficiency of Small Buffers



The optimal scheduling assignment for a multi-hop line network with 10 relay nodes.

- Optimal spatial reuse parameter is $m = 3$.
- Interference induces a natural spacing between packets.
- ALOHA with contention probability 1 is optimal.
- Small buffers (as low as unit size) are sufficient for the optimal operation of the network.

Benefits of Keeping Unit Buffer Sizes

- Scheduling is simple and completely decentralized: nodes having a packet transmit, others remain idle.
- Since packets never stack-up, average end-to-end delay is kept low.
- Generally requires buffering at the source. However, delay incurred after a packet leaves the source is much more tightly controlled.
- Large buffers increase hardware cost and energy consumption.

A Revised Transmission Scheme for Ad Hoc Networks

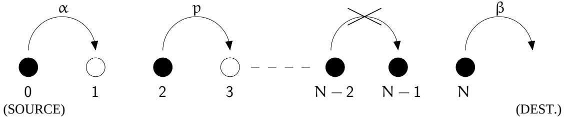
The Three Rules:

1. All the buffering is performed at the source node (relay nodes have unit buffer sizes).
2. Transmissions are not attempted by nodes if their adjacent node's buffer already contains a packet.
3. Packets are retransmitted until they are successfully received.

- This scheme is completely distributed and helps regulate the flow of packets in the network by artificially spacing packets.
- Transmissions can occur only if node has a packet and adjacent node has none - analogous to the TASEP!

A Review of TASEPs with Open Boundaries

- The TASEP models the dynamics of self-driven systems with several interacting particles; is a paradigm for non-equilibrium systems † .
- Particles are injected into the system at site 0, and they hop rightward until they exit the system at site N.



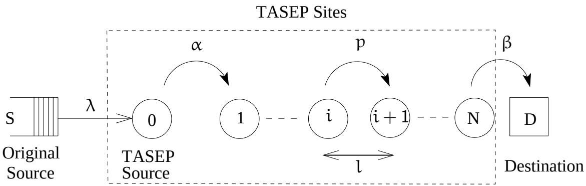
Snapshot of the TASEP system model along with the hopping probabilities. Filled circles indicate occupied sites and the rest indicate holes.

- Note the analogies:

- Sites ⇔ Nodes.
- Particles ⇔ Packets.
- Exclusion principle ⇔ Unit buffer sizes.
- Hopping probability ⇔ Link reliability.

†N. Rajewsky et al., "The asymmetric exclusion process: comparison of update procedures," *Journal of Statistical Physics*, 1998.

r-TDMA-based Line Network



The wireless line network is modeled as a source node with a large buffer connected to the TASEP particle flow model.

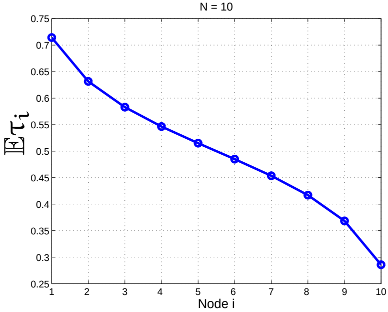
- Regular line network with spacing l.
- MAC scheme: randomized TDMA (r-TDMA): Transmitting node is chosen uniformly randomly in each time slot $\Leftrightarrow$ random sequential TASEP.
- Channel: PLE γ, Rayleigh fading, Noise N_0.
- Since interference is absent in the system, the success probability across each link is $p_s = \mathbb{P}(\text{SNR} > \Theta) = \exp(-\Theta N_0 l^{-\gamma})$.
- We primarily study the steady state behavior (in the long-time limit).

r-TDMA-based Line Network: Steady State Occupancies

- Let τ_i denote the occupancy of node i, $1 \leqslant i \leqslant N$.

$$\mathbb{P}(\tau_i = 1) = \mathbb{E}\tau_i = \frac{1}{2} + \frac{1}{4}\frac{(2i)!}{(i!)^2}\frac{(N!)^2}{(2N+1)!}\frac{(2N-2i+2)!}{[(N-i+1)!]^2}(N-2i+1).$$

- The occupancies are independent of p_s!



Notice the particle-hole symmetry. $\mathbb{E}\tau_i = 1 - \mathbb{E}\tau_{N+1-i}$.

r-TDMA-based Line Network: Steady State Throughput

- At steady state, rate of packet flow is a constant.
- We have analytically derived that

$$T = \frac{p_s(N+2)}{2(N+1)(2N+1)}.$$

- The system throughput at steady state is proportional to the link reliability and upper bounded by $p_s/4$.
- It decreases with system size. $T \sim p_s/(4N)$ for $N \gg 1$.
- **Interpretation:** No spatial reuse; both the transmitting and receiving nodes' buffers are each occupied independently with probability $1/2$.

r-TDMA-based Line Network: Steady State Delays

Theorem

The delay experienced by a packet at node i, D_i *follows a geometric distribution with mean*

$$\mathbb{E}D_i = \frac{2(N+1)(2N+1)\mathbb{E}\tau_i}{(N+2)p_s}, \quad 0 \leqslant i \leqslant N.$$

Also, the average end-to-end delay is given by

$$\mathbb{E}D_{e2e} = \frac{2N^2+3N+1}{p_s}.$$

The three events that need to occur in the following order for the packet to be able from an arbitrary node i to hop to node $i+1$ successfully are

(1) Node $i+1$ has an empty buffer.

(2) Node i is picked for transmission.

(3) Node i's transmission is successful.

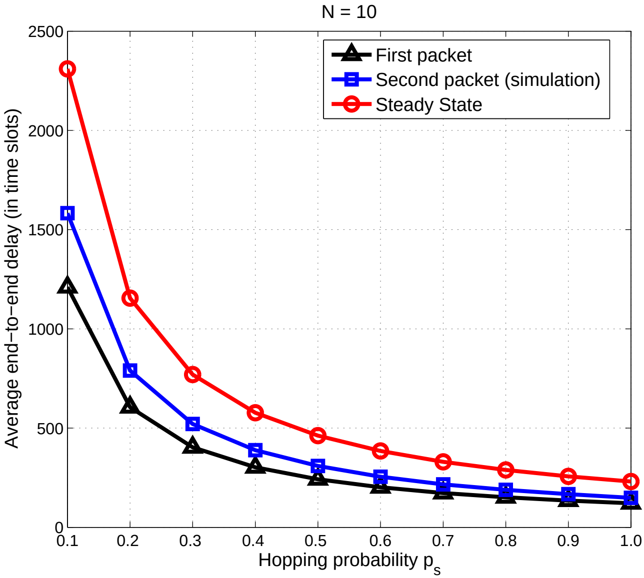
r-TDMA-based Line Network: First Packet's Delay

Corollary

The end-to-end delay experienced by the first transmitted packet follows a negative binomial distribution with parameters $N+1$ *and* $p_s/(N+1)$.

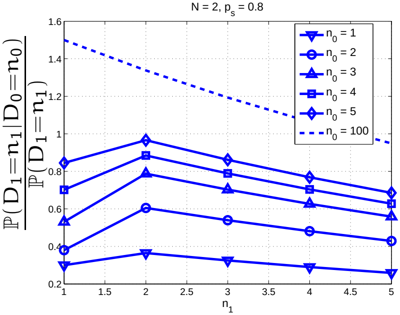
- Basic, yet useful result on the network's transient behavior.
- The average end-to-end delay of any packet is lower- and upper-bounded by the average end-to-end delay for the first packet and the average end-to-end delay at steady state respectively.
- $$\frac{\mathbb{E}D_{\mathrm{e2e}}}{\mathbb{E}D^{\mathrm{f}}_{\mathrm{e2e}}} = \frac{N+1}{2N+1} \sim \frac{1}{2}.$$

r-TDMA-based Line Network: Mean Delay Bounds



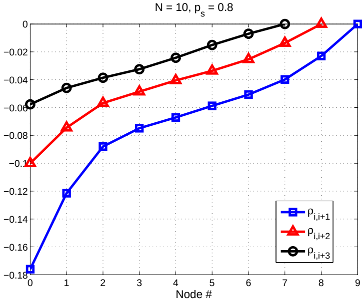
r-TDMA-based Line Network: Delay Correlations

- We have analytically derived the spatial delay correlations in a network with 2 relays, and our procedure can be extended for larger networks.
  1. D_2 is independent of the other delays. (so is D_N, in a network with N nodes.)
  2. D_0 and D_1 are strongly correlated.



The conditional probabilities indicate the strong correlations.

r-TDMA-based Line Network: Delay Correlations (Contd.)

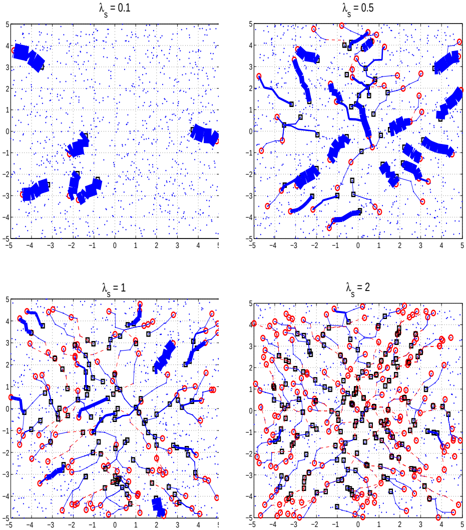


- All the delay correlation coefficients ρ_{ij} are non-positive.
- The delay correlations across nodes farther apart and closer to the destination are seen to be relatively light.
- The delay variance is dominated by the first few nodes (nodes closer to the source).

Part 3: Extensions and Proposed Research

Proposed Research

- Extend results from the line network to a system of line networks.
- Characterize the throughput-delay-reliability (TDR) region.
- Use statistical mechanics tools to understand the dynamics of CSMA-based networks.
- Study the transient behavior of multi-hop ad hoc networks.
- Consider different topologies and bidirectional traffic.



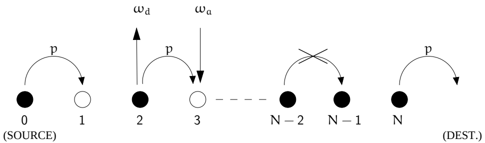
- Source nodes: PPP(λ_s).
- Overall: PPP ($\lambda = 10$).
- Intra-route MAC: r-TDMA.
- Routing: Nearest neighbor in a sector of angle 90^0.
- ○: source nodes.
- □: destination nodes.
- The line width is proportional to the no. of successful transmissions across that link.

Question 1: Routing Strategies in Ad Hoc Netw. (Contd.)

- What value of λ_s optimizes the sum throughput of the system?
- How does one choose the value of the routing angle ϕ?
- Instead of routing over nearest neighbors, is it beneficial to hop over farther (say m) neighbors?
- How to jointly optimize λ_s, ϕ and m for a given MAC and routing scheme?
- If all sources have the same message to transmit, how fast can the system be flooded? What are the properties of the paths formed?

Question 2: TDR Characterization of Line Networks

- When reliability is set to 100%, the TDR region is a hyperbolic curve (the product of throughput and delay is a constant).
- When the link reliability is low, packets will be retransmitted several times, and this results in a large queueing delay.
- Instead, one may choose to drop packets with a certain probability q, while maintaining the throughput at a reasonably good value.
- Use ideas from TASEP coupled to Langmuir Kinetics? $(\omega_d = q, \omega_a = 0)$

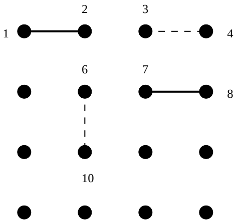


ω_a and ω_d are the (site-independent) absorption and desorption rates.

- Related questions: How does the TDR region change if q is site-dependent or queue state-dependent?

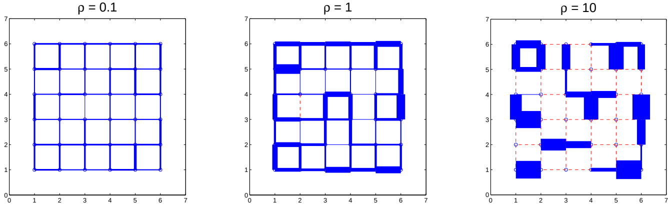
Question 3: Statistical Mechanics of CSMA Networks

- Consider nodes arranged on a Manhattan lattice with unit grid length.
- The transmission ranges of each node is unity, and the channel access scheme is CSMA.
- A link may become active only if none of the two nodes' neighbors are active.
- Assume that admissible links get activated at unit rate.
- Once a link is formed, it deactivates at rate $1/\rho$.



When nodes 1 and 2 form an active link, nodes 3 and 4 or nodes 6 and 10 cannot establish links with each other.

Question 3: Stat. Mechanics of CSMA Networks (Contd.)



The tradeoff between ρ and fairness is evident from the plots.

- What is the optimum ρ? How does the optimum ρ depend on the degree of the nodes?
- How long does it take for a packet to travel between arbitrary nodes i and j?
- It will be useful to explore the relationship between this model and the TASEP.

Summary

- Introduced two new tools for ad hoc network analysis.
  - Distance distributions.
  - The TASEP.
- Analytically derived internode distances in an isotropic BPP, and used it to study single-hop properties of wireless networks.
- Characterized the average end-to-end delay and throughput in multi-hop line networks using tools from statistical mechanics, while explicitly taking into account the delay correlations.
- More concepts from stochastic geometry and statistical mechanics must be explored in order to help enhance our understanding of wireless networks.