The TASEP Applied to the Throughput and Delay Analysis of Line Networks

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Overview

- Draw an analogy between the totally asymmetric simple exclusion process (TASEP) and simple multihop line networks.
- Tap into the rich theory of TASEP and its results to analyze
  - Steady state configurations and their probabilities.
  - Dynamic behavior of the multihop line network.
  - Characterize the throughput-delay-reliability tradeoff.

A Simple Buffering Scheme for Line Networks

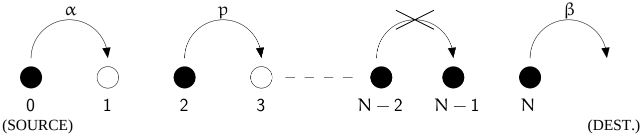
- Buffering is pushed back to the source, while relay nodes are essentially bufferless (have buffer sizes of unity).
- Nodes do not accept incoming packets if their buffer is already full.
  1. Simple way to prevent packets from getting too close.
  2. Self-organization :
     - Transmitting nodes are at least two hops apart.
     - The exclusion principe regulates the traffic injected in a backpressure-like manner.
- Single-buffer multihop networks $\Leftrightarrow$ TASEP.

TASEP with Open Boundaries

- A topic in statistical mechanics.
- Used to describe the dynamics of self-driven systems with several interacting particles.
- Applied in problems such as
  - Traffic flow modeling.
  - Kinetics of bipolymerization.
  - Stock market fluctuations.
- Regarded as a paradigm for non-equilibrium systems.

TASEP Model

- The source node is numbered 0 and there are N relay nodes.
- Configuration of the sites : $\tau_i \in \{0, 1\}$ - occupied or not.
- Hopping between sites is possible only if the configuration $\{\tau_i, \tau_{i+1}\}$ is $\{1,0\}$.

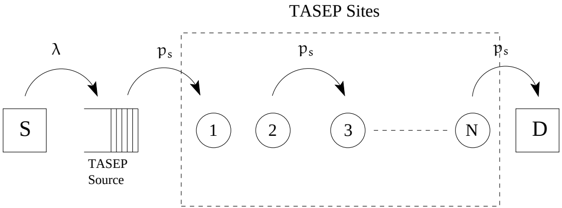


A snapshot of the TASEP system model along with the hopping probabilities. Shaded circles indicate occupied sites.

- Exclusion principle creates a particle-hole duality.

A simple MAC scheme in Line Networks

IDEA :



The actual source is connected to a flow regulator.

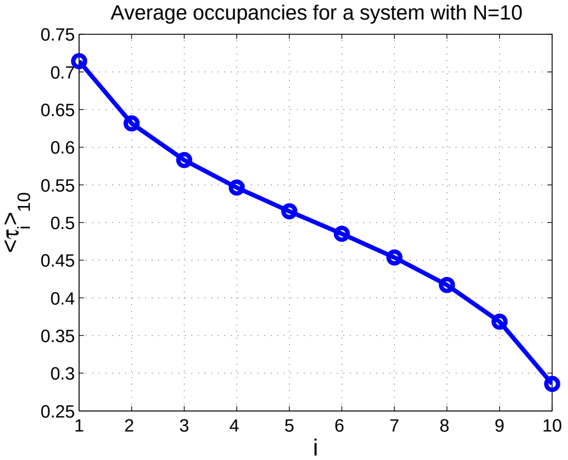
- p_s : Channel success probabilities.
- TASEP with random sequential update.
  - At each time step, a site is randomly picked w.p. $1/(N+1)$ and hopping is performed on it.

Steady State Analysis

- Assume that the TASEP source buffer is always non-empty.
- Use the matrix product ansatz (MPA) as an analysis tool.
- Steady state occupancies :
  - Fraction of time each site is occupied at steady state.
  - Queueing theory : utilization factor of the server.

$$\langle \tau_i \rangle_N = \frac{1}{2} + \frac{1}{4} \frac{(2i)!}{(i!)^2} \frac{(N!)^2}{(2N+1)!} \frac{(2N-2i+2)!}{[(N-i+1)!]^2} (N-2i+1).$$

Steady State Analysis (Contd.)



The steady state occupancies are independent of p_s. However, lower the value of p_s, longer it takes to get to steady state.

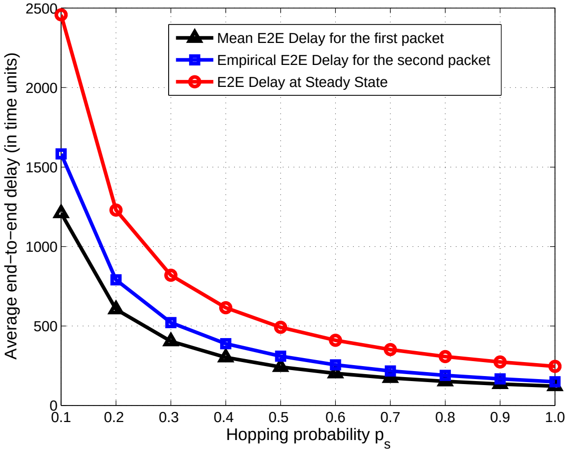
Delay Dynamics

- Steady State :
  - Delay at the TASEP source: $D_0 \sim \textbf{Geo}\left(\frac{p_s(1-\langle\tau_1\rangle)}{(N+1)}\right)$.
  - Delay at the relay nodes: $D_i \sim \textbf{Geo}\left(\frac{p_s(1-\langle\tau_{i+1}\rangle)}{N+1}\right)$.
  - Delay at the final relay node: $D_N \sim \textbf{Geo}\left(\frac{p_s}{N+1}\right)$.
  - Mean e2e delay:

$$\mathbb{E}[D_{e2e}] = \frac{N+1}{p_s}\left[1 + \sum_{i=1}^{N} \frac{1}{1-\langle\tau_i\rangle}\right].$$

- First Packet :
  - $D_i \sim \textbf{Geo}\left(\frac{p_s}{N+1}\right)$ and are independent.
  - End-to-end delay: **Neg. Bino**$\left(N+1, \frac{p_s}{N+1}\right)$.
- Waiting time in the buffer depends on the source traffic model.

Delay vs Link Reliability



The average end-to-end delay at steady state and the same for the first two transmitted packets.

Steady State Throughput

- At steady state, rate of flow of particles over each link is the same.
- Current through link $i = \langle \tau_i (1 - \tau_{i+1}) \rangle$.
- Using the MPA, the throughput is expressed as

$$T = \frac{p_s(N+1)}{2(2N+1)}.$$

- Throughput reduces as the system size increases.
- $p_s/4 < T \leqslant p_s/3$.
- Throughput is proportional to the link reliability.

Extensions

- How do the dynamics of the system change when some links are less reliable than the others ?
- Look at more complex topologies e.g. mesh networks.
- Study delay correlations.
- Relationship to backpressure algorithms ?
- What lies ahead ?
  - Different updating procedures correspond to different MAC schemes.
    - Parallel TASEP $\Leftrightarrow$ ALOHA.
    - Sub-lattice parallel TASEP $\Leftrightarrow$ m-TDMA.
  - How to characterize the optimum spatial reuse ?