Simplified Analysis and Design of MIMO Ad Hoc Networks

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Introduction

- An ad hoc network's performance is severely susceptible to path loss, fading and interference.
- Using multiple antennas at each node mitigates the effect of fading.
- However, design of MIMO ad hoc networks becomes a challenging problem, especially as network size increases.
- The 'erristor' framework is useful in characterizing transmissions and leads to simplified analysis and design.

Outline

- Introduction to a simple, yet powerful concept: *erristor*.
- Extend the formalism to multihop MIMO systems.
- How does this help in analyzing/modeling MIMO ad hoc networks ?
- Provide a simple example.
- Superiority of MIMO over SISO systems at high SNR.

System Model and Assumptions

- A multihop MIMO network with m antennas at each node.
- With a transmit power P, each antenna transmits at power P/m (No CSI at the transmitters).
- Transmitter aims at diversity maximization.
- Channel effects - path loss (with exponent α) and flat (narrow band) block Rayleigh fading.
- Perfect MAC scheme or light traffic analysis.
- Selection Combining.

The Erristor Framework

Consider a SISO link.

- Transmission is successful if SNR at receiver is greater than Θ.
- The reception (or success) probability p_r over a link of distance d at a transmit power P_0 and noise variance N_0 is given by

$$p_r = \exp(-\Theta N_0 / P_0 d^{-\alpha}).$$

- Denote $R := \Theta N_0 / P_0 d^{-\alpha}$ (normalized mean noise-to-signal ratio (NSR)) as an erristor and its value as erristance [†] .

[†] M. Haenggi, "Analysis and design of diversity schemes for ad hoc wireless networks," *IEEE J. Selected Areas Commun.*, vol. 23, pp. 19-27, Jan. 2005.

The Erristor Framework

Why the name 'erristor' ?

1. For $R \ll 1$, $R \approx 1 - p_r$, the packet loss (error) probability.
2. Over a n-hop serial route, the end-to-end reliability is

$$p_{EE} = \exp(-\sum_{i=1}^{n} R_i) = \exp(-R_{tot}),$$

where the sum of R_i's can be replaced by an equivalent R_{tot}. Notice the resistor-like series connection property.

- The erristor formalism permits the mapping of unwieldy probability expressions to a simple circuit-like framework.

Extension of the Erristor Formalism

Consider a MIMO point-to-point link.

- Received power Q at each antenna is a chi-square distributed RV with $2m$ degrees of freedom and mean $\bar{Q} = P_0 d_{ij}^{-\alpha}$.

$$F_Q(q) = 1 - e^{-(qm/\bar{Q})} \sum_{k=0}^{m-1} \frac{1}{k!} \left(\frac{qm}{\bar{Q}} \right)^k, \quad q \geqslant 0.$$

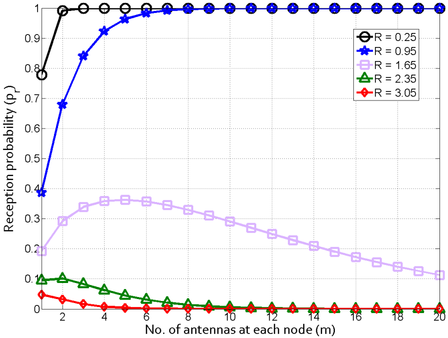
- Selection combining strategy picks $S = \max\{Q_1, \ldots, Q_n\}$ for decoding.
- Reception probability is given by $\Pr[S \geqslant \Theta N_0]$.

$$p_r = 1 - \left(1 - e^{-\Theta N_0 m/\bar{Q}} \sum_{k=0}^{m-1} \frac{1}{k!} \left(\frac{\Theta N_0 m}{\bar{Q}} \right)^k \right)^m.$$

Extension of the Erristor Formalism

- With R as the normalized mean NSR, we get

$$p_r = 1 - \left(e^{-Rm} \sum_{k=m}^{\infty} \frac{1}{k!} (Rm)^k \right)^m .$$



Notice the contrast in asymptotic behavior as one set of curves approach 1, while the others tend to 0.

Asymptotic Behavior

- Poisson$(\lambda) \approx \mathcal{N}(\lambda, \lambda)$ for $\lambda \gg 1$

$$p_r \approx 1 - \left(\frac{1}{\sqrt{(2\pi R m)}} \int_m^\infty e^{-\frac{(k-Rm)^2}{2Rm}} dk \right)^m .$$

- Writing in terms of the Q-function,

$$p_r \approx 1 - \left(Q\left(\frac{m(1-R)}{\sqrt{Rm}} \right) \right)^m .$$

- To study the behavior as $m \to \infty$, use

$$Q(x) \leqslant \frac{1}{x\sqrt{2\pi}} e^{-x^2/2}, \quad x > 0.$$

Asymptotic Behavior

- $R < 1$

$$p_r \gtrapprox 1 - \left(\frac{\sqrt{R}}{(1-R)\sqrt{2m\pi}} \right)^m e^{-m^2(1-R)^2/2R}.$$

  $p_r \to 1$ as $m \to \infty$

- $R > 1$

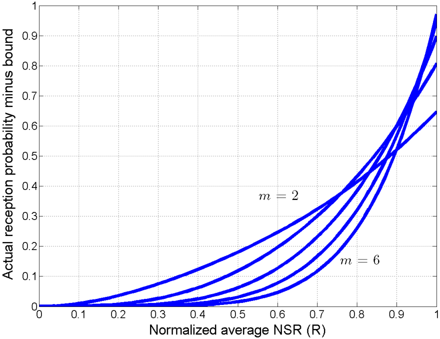
$$p_r \lessapprox 1 - \left(1 - \frac{\sqrt{R}}{(R-1)\sqrt{2m\pi}} e^{-m(R-1)^2/2R} \right)^m.$$

  $p_r \to 0$ as $m \to \infty$

- *Phase transition* occurs at $R = 1$ (average SNR of Θ).

Markov Approximation

- Simplify the cumbersome expression using the Markov tail approximation to obtain $p_r \geqslant 1 - R^m$ (See Figure).



The Markov approximation for p_r is tight at high SNR values.

- $p_r \approx e^{-R^m}$ at high SNR or large m.
- R^m is the erristance for the MIMO link.

Design of MIMO Ad Hoc Networks

The design problem:

How to choose link erristances such that p_{EE} is at least at the desired level p_D ?
- Requires knowledge of erristor equivalents.

Series Connection (Multihop Connection)

- Reception probabilities multiply; $p_{EE} = e^{-\sum_{i=1}^{n} R_i^m}$.
- Equivalent erristance is $R_{tot} = \sum_{i=1}^{n} R_i^m$.

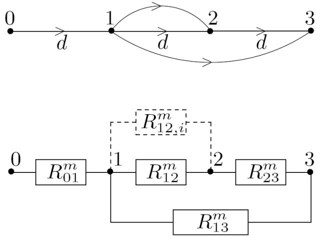
Parallel Connection

- Time and path diversity, cooperative and implicit transmissions.
- Equivalent erristance is bounded as $R_{tot} \lessapprox \prod_{i=1}^{n} R_i^m$ † .

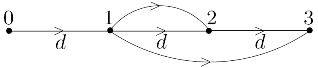
Parallel and series equivalents help simplifying most networks.

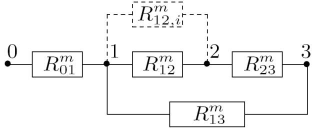
† M. Haenggi, "Analysis and design of diversity schemes for ad hoc wireless networks," *IEEE J. Selected Areas Commun.*, vol. 23, pp. 19-27, Jan. 2005.

A Three Hop MIMO Network



- Each node has m antennas.
- Node 1 transmits its packet twice, once to node 2 and once over the link 1→3.
- Node 2 overhears transmission from $1 \to 3$, and implicitly knows 1's packet.
- **Requirement** : $p_D = 0.9 \Leftrightarrow R_{tot} = -\ln(p_{EE}) \leqslant 0.105$.





Recall that R is inversely proportional to P, $d^{-\alpha}$.

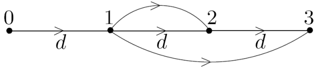
Scenario 1: Each node expends the same net transmission power.

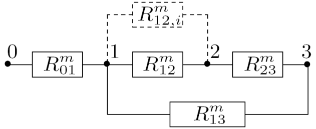
- $R_{12,i} = 2^{-\alpha} R_{13}$ since $d_{13} = 2d_{12}$.
- $R_{01} = R_{23} = R(\text{say})$, because $d_{01} = d_{23}$.
- Node 1 needs to transmit at the same power

$$\frac{d^\alpha}{R} = \frac{d^\alpha}{R_{12}} + \frac{(2d)^\alpha}{R_{13}}.$$

- Possible setting: $R_{13} = 2^\alpha R_{12}$, which gives $R_{12} = 2R$.

A Three Hop MIMO Network



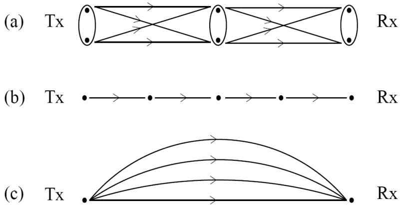


- $R_{tot} = R^m + ((2R)^{2m} + R^m)(2^\alpha 2R)^m \leqslant 0.105$.
  At $\alpha = 3.5$, $R = 0.048$ is a solution for $m = 1$.
  For $m = 3$, $R = 0.143$ ($\approx 67\%$ reduction in power).

Scenario 2: Node 2 exhausts its battery.

- Link $1 \rightarrow 2$ becomes useless.
- The erristor network consists of just R^m_{01} and R^m_{13} in series.
- Resources need to be reallocated to these nodes only.

Comparison of MIMO with SISO schemes



- Apply the erristor framework to compare the following transmission schemes.
  - a) The MIMO multihop scheme.
  - b) The SISO multihop scheme.
  - c) The SISO system with retransmission involved.
- Assume same number of total transmissions and the same p_D for each scheme.
- Study the normalized energy consumption (per packet sent) and how it varies depending on p_D.

Comparison of MIMO with SISO schemes

- With n transmitting nodes and m outgoing paths from each node, the normalized energy consumption (per packet) is

$$E_{tot} = \sum_{i=1}^{n} \sum_{j=1}^{m} \frac{d_{ij}^{\alpha}}{R_{ij}}.$$

a) MIMO multihop: $$E_{tot} = mnd^{\alpha} \left(\frac{1}{n} \right)^{\alpha} \left(\frac{n}{R_{tot}} \right)^{\frac{1}{m}}.$$

b) SISO multihop: $$E'_{tot} = mnd^{\alpha} \left(\frac{1}{mn} \right)^{\alpha} \frac{mn}{R_{tot}}.$$

c) SISO (retransmission): $$E''_{tot} = mnd^{\alpha} \left(\frac{1}{R_{tot}} \right)^{\frac{1}{mn}}.$$

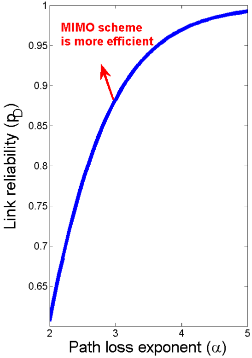
MIMO vs SISO Multihop

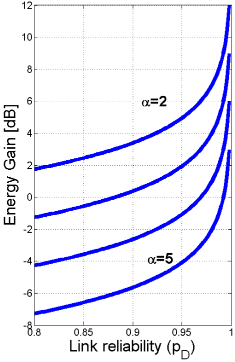
Consider the case $m = 2$ and $n = 2$.

$$\frac{E_{tot}}{E'_{tot}} = 2^{\alpha - \frac{3}{2}} R_{tot}^{\frac{1}{2}}.$$

MIMO is more energy efficient than the SISO multihop scheme if

$$R_{tot} < 2^{3-2\alpha} \Leftrightarrow$$
$$p_D > e^{-2^{(3-2\alpha)}}.$$





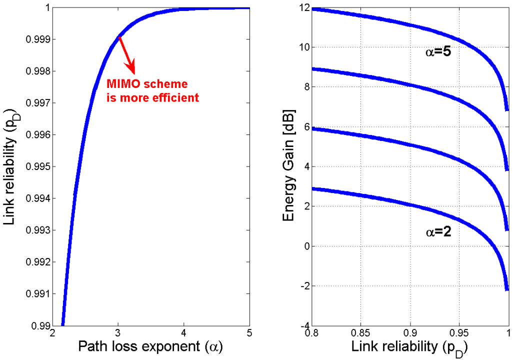
Substantial energy gains are observed as $p_D \to 1$.

MIMO vs SISO Time Diversity

$$\frac{E_{tot}}{E''_{tot}} = 2^{-\alpha+\frac{1}{2}} R_{tot}^{-\frac{1}{4}}.$$

MIMO is better than the SISO time diversity scheme when

$$R_{tot} > 2^{2-4\alpha} \Leftrightarrow$$
$$p_D < e^{-2^{(2-4\alpha)}}.$$



For practical purposes, MIMO is more energy efficient.

Concluding Remarks

- The erristor concept greatly simplifies analysis and design problems for MIMO ad hoc networks employing selection combining.
- Resource (re)allocation problems can be reduced to simple polynomial equations.
- Based on the erristor framework, MIMO is known to outperform SISO, especially at high SNR values.
- Asymptotic behavior of the MIMO network is studied, and a critical value of SNR at which phase transition occurs is calculated.